A Pre-Bohr Explanation of the Periodic Table: How Could a Wrong Theory Fit Data So Well?

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1. Formulation of the problem

- Usually, a good fit between a theory and the data means that the theory is true.
- However, there was a known exception.
- In the early 20 century, John Nicholson showed that:
  - the atomic weight w of each element from the periodic table can be represented – with accuracy 0.1
  - as an integer combination of 4 basic weights: $w_1 = 0.51282$, $w_2 = 1.008$, $w_3 = 1.6281$, and $w_4 = 2.3615$:

    $$w = n_1 \cdot w_1 + n_2 \cdot w_2 + n_3 \cdot w_3 + n_4 \cdot w_4 \text{ for some integers } n_i \geq 0.$$
- This led him to a conclusion that all atoms consist of combinations of 4 basic particles with these weights.
- The fit was perfect – but the theory turned out to be wrong.
- How can it be?

2. What is known and what we do

- In our previous paper, we have shown that for the specific 4 weights w_i selected by Nicholson:
  - *any* number larger than $n_0 = 3.03$ – and not just the atomic weights
  - can be represented, with accuracy 0.1, as an integer combination of these 4 weights.
- In this talk, we show that a similar property – maybe for some other n_0 – holds for any selection of small random weights w_i.
- Specifically, we show it on a toy example when all 4 selected weights w_i are close to 1.
- Similar arguments – with a different n_0 – work when we have some values w_i close to 2.

3. Our explanation

- To get the overall weight close to an integer n, we need to add n values which are close to 1.
- So, we must have $n_1 + n_2 + n_3 + n_4 = n$, where n_i is the number of times we pick w_i.
- Each tuple (n_1, n_2, n_3, n_4) can be graphically described if we:
  – place n 1s in a row, and
  – add dividers after the first n_1 1s, after $n_1 + n_2$ 1s, and after $n_1 + n_2 + n_3$ 1s.
- This way, possible tuples are in 1-1 correspondence with selecting 3 numbers out of $n + 3$.
- So, there are $N = \binom{n+3}{3} = \frac{(n+3)\cdot(n+2)\cdot(n+1)}{1\cdot 2\cdot 3}$ such tuples.

4. Our explanation (cont-d)

- Since we picked 4 random weights w_i, it is reasonable to conclude that we have N numbers randomly distributed around n.
- We have no reason to assume that some values are more probable or that different sums are dependent.
- So, it makes sense to assume:
  - that all values are equally probable and independent, i.e.,
  - that we have N independent uniformly distributed random variables.
- For each value $w \approx n$:
  - the only possibility of not being approximated with accuracy 0.1 by one of the N sums
  - when all N sums lie outside the interval $[w - 0.1, w + 0.1]$ of width 0.2.

5. Our explanation (cont-d)

- The probability for each sum to be outside this interval is equal to

$$1 - 0.2 = 0.8.$$

- So the probability for all N sums to be outside is 0.8^N.
- To get this probability smaller than 0.05, we need $0.8^N \leq 0.05$, i.e., $N \geq 13.4$, which happens for $n \geq 3$.
- So in this case $n_0 = 3$.
- If we want the probability smaller than 0.1%, we need $N \geq 30.9$, which happens starting with $n = n_0 = 4$.

6. Reference

- O. Kosheleva and V. Kreinovich, "Why was Nicholson's theory so successful: an explanation of a mysterious episode in 20 century atomic physics", *Mathematical Structures and Modeling*, 2021, Vol. 60, pp. 39–43.

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