Why the Simplest or the Most Beautiful Solution Is Often the Best

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1. Formulation of the problem

- How do physicists come up with equation that describe nature – i.e., that provide the best fit for observations?
- Some of them look for the simplest of possible equations.
- Some look for the most beautiful equations.
- And somehow all of them come up with exactly the equations that bet fit the data.
- These are different criteria.
- In general, what is simpler is not necessarily more beautiful, and vice versa.
- However, many different optimality criterion do lead to the exact same result.
- In this talk, we provide a possible explanation for this rather mysterious fact.

2. Symmetry: general idea

- Why can we make predictions about the world?
- Because many situations are similar, so:
  - if we encounter a new situation which is similar to the one we experienced earlier,
  - it is reasonable to predict that the outcome of the new situation will be similar.
- In physics, this similarity is formalized as *symmetry* – when changing the situation in a certain way does not change the outcome.

3. A simple example

- Numerical value of a physical quantity depends on the choice of a measuring unit.
- If we use cm instead of m, numerical values change but the quantities remain the same.
- In general:
  - if we replace the original unit with a c times smaller one,
  - all numerical values are multiplied by c: $x \mapsto T_c(x) = c \cdot x$.
- For each dependence $y = f(x)$, the numerical value of y can also change to $C \cdot y$ for some C.
- To eliminate dependence on the y-unit, we can consider the whole family $\{C \cdot f(x)\}_C$.

4. What we mean by the best

- The best – optimal – means that:
  - we have a way to compare two families, and
  - we select the family $a_{\rm opt}$ which is better than all others:

$$\forall a\,(a_{\rm opt}\succeq a).$$

5. We should consider final optimality criteria

- What if several families are the best in this sense?
- Then, we can use this non-uniqueness to optimize something else.
- So, in the final optimality criterion, there is only one optimal family.

6. This leads to an explanation

- It is reasonable to assume that the relative quality should be invariant under scaling transformation $T_c : \{C \cdot f(x)\}_C \mapsto \{C \cdot f(c \cdot x)\}_C$:
  - if $a \succeq b$
  - then $T_c(a) \succeq T_c(b)$.
- Our main result is that:
  - for *any* final scale-invariant optimality criterion $\succeq$,
  - the optimal family is the one which is invariant under scaling.
- Indeed, since a_{opt} is the best, we have $a_{\text{opt}} \succeq T_{1/c}(a)$ for all a.
- Thus, since $\succeq$ is scale-invariant, we get $T_c(a_{\text{opt}}) \succeq a$ for all a.
- This means that the family $T_c(a_{\text{opt}})$ is also optimal.
- However, since the criterion $\succeq$ is final, there is only one optimal family.
- So indeed, $T_c(a_{\text{opt}}) = a_{\text{opt}}$.
- For scale-invariance, this means that $a = \{C \cdot x^a\}$ for some a.

7. This leads to an explanation (cont-d)

- So, no matter what optimality criterion we use:
  - as long as this criterion is scale-invariant,
  - we get the same class of optimal functions.
- We showed it on the example of scaling.
- However, this argument works for all possible symmetries.
- This explains why optimizing different criteria often leads to the same solution.

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