Reversible and quantum computing involving random processes: local time naturally appears

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1. Reversible computing: why

- One of the big problems of modern computing is that computers use a large amount of energy, up to 10% of the world's consumption.
- And this portion increases.
- There are technological reasons for this consumption, but there is also a fundamental physical reason.
- According to thermodynamics, irreversible processes increase entropy and thus, cause heat release.
- And computing uses a lot of irreversible processes.
- For example, an and-gate transforms two signals a and b into a single signal $c = a \,\&\, b$.
- When $c = 0$, we cannot uniquely reconstruct the inputs (a, b); it could be $(0, 0)$, or $(0, 1)$, or $(1, 0)$.

2. Reversible computing: why (cont-d)

- To decrease computers' energy consumption (and heating the environment), it is desirable to make computations reversible.
- Limitation to reversible computing is also needed for quantum computing.
- Indeed, there computation is done on the level of elementary particles – and on this level, all physical processes are reversible.

3. Reversible computing: how

- To make computations reversible, a natural idea is to add auxiliary inputs.
- For example, we can make an and-gate reversible if we add an auxiliary input y and transform (a, b, y) into $(a, b, y \oplus (a \,\&\, b))$.
- Here $\oplus$ denoted exclusive "or" (i.e., addition modulo 2).
- Similarly:
  - to make computations $x \mapsto f(x)$ with real numbers reversible,
  - we can add additional input y and transform (x, y) into $(f(x), g(x, y))$ for some $g(x, y)$.
- Here, reversible means that the number of output states be equal to the number of input states.

4. Reversible computing: how (cont-d)

- If we compute with accuracy ε, then the number of input states is proportional to the area of the (x, y) domain.
- In these terms, reversible means area-preserving, and this leads to $g(x, y) = y/f'(x)$, where $f'(x)$ means the derivative.

5. What if we want to simulate a random process?

- Many real-life processes are random.
- It is therefore important to simulate them.
- In these simulations, just like in all other simulations, it is desirable to use reversible and quantum computing.
- For this purpose, we need to replace possibly irreversible computations $t \mapsto x(t)$ with reversible ones $(t, y) \mapsto (x(t), g(t, y))$ for some function $g(t, y)$.
- For smooth processes, as we have mentioned, we should take

$$g(t, y) = \frac{y}{x'(t)}.$$

- So, we need to extend this expression to general – not necessarily smooth – processes.
- Interestingly, such an extension, in effect, already exists.

6. What if we want to simulate a random process (cont-d)

- To be more precise, $1/x'(t)$ is a smooth case of *local time* $\ell(t)$ which is defined as follows.
- For every x-interval $I = [x, x+\varepsilon]$, we consider the overall duration $d(\varepsilon)$ of all the time intervals at which we had $x(t) \in I$.
- Then, we define $\ell(t)$ as the limit of the ratio $d(\varepsilon)/\varepsilon$ when $x = x(t)$.
- It is known that in the smooth case, $\ell(t) = 1/x'(t)$.
- It is known in the general case:
  - when we have a sequence of smooth processes approximating a given one,
  - the resulting values $1/x'(t)$ tend to $\ell(t)$.
- Thus, in general, we can use local time to have a reversible or quantum simulation of a random process $x(t)$ as $(t, y) \mapsto (x(t), y \cdot \ell(t))$.

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