Brans-Dicke Scalar-Tensor Theory of Gravitation May Explain Time Asymmetry of Physical Processes

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1. Observable Time Asymmetry: A Problem

- Most equations of fundamental physics are time symmetric:
  - starting from the ordinary differential equations (e.g., the classical Newton's equations of motion)
  - to partial differential equations describing physical fields like electromagnetism or gravitation.
- So, if we simply reverse the direction of time t, the resulting fields will satisfy the same diff. equations.
- From this viewpoint, a time reversal of a physically reasonable process should also be physically reasonable.
- In practice, many physical processes are not reversible:
  - if we drop a fragile cup, it will break into pieces;
  - however, a broken cup cannot get together to form a whole cup.

2. How This Problem Is Explained Now

- The problem of time asymmetry is known since Bolzmann's 19th century work on statistical physics.
- In modern physics, this problem is usually resolved by assuming that the initial conditions are *random*.
- *Problem:* this randomness assumption is outside the usual PDE formulation of physical equations.
- It is therefore desirable to come up with an alternative explanation within the PDE framework.
- We show that the equations of scalar-tensor theories of gravitation are, in some sense, *not* T-symmetric.
- This may explain observed time asymmetry.

3. General Relativity: Reminder

- In general, the field equations of a physical theory correspond to the minimum of the action

$$S = \int L\sqrt{-g}\,dt\,dV, \text{ where } g = \det(g_{\alpha\beta}).$$

- In particular, for the General Relativity theory (GRT):

$$L_{GRT} = \frac{1}{16\pi G} R + L_{\text{mat}}, \text{ where}$$

  - G is the gravitation constant,
  - L_{mat} is the Lagrangian of matter,
  - $R \stackrel{\text{def}}{=} g^{\alpha\beta} R_{\alpha\beta}$ is the Ricci scalar,
  - $R_{\alpha\beta} \stackrel{\text{def}}{=} R^{\gamma}_{\alpha\gamma\beta}$, and
  - $R^{\delta}_{\alpha\gamma\beta}$ is the curvature tensor.
- Varying over $g_{\alpha\beta}$, we get $R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi G T_{\alpha\beta}$, where $T_{\alpha\beta}$ is the matter's energy-momentum tensor.

4. Motivations for Modifying General Relativity

- The observed gravitational acceleration a is often much larger that what follows from the observable mass $M_{\rm obs}$:
$$a \gg \frac{G M_{\rm obs}}{r^2}.$$
- *Traditional solution:* there are non-observable masses ("dark matter", "dark energy").
- *Problem:* 95% of the mass is "dark matter" and "dark energy".
- *Alternative idea:* maybe the gravitational "constant" G is different at different locations, i.e., is a new field.
- In such a theory, to describe gravitation, we need both the metric field $g_{\alpha\beta}$ and the new scalar field $\varphi \stackrel{\rm def}{=} \dfrac{1}{G}$.
- The corresponding scalar-tensor theory of gravitation was indeed proposed by Brans and Dicke.

5. Brans-Dicke Theory: Reminder

- In terms of this new field, the Einstein's term $\frac{1}{G}R$ from the Lagrangian takes the form φR.
- We also need to add the effective energy density $\frac{\varphi_{,\alpha}\varphi^{,\alpha}}{\varphi}$ of the scalar field, so we get:

$$L_{\text{BDT}} = \varphi\left(R - \omega \cdot \frac{\varphi_{,\alpha}\varphi^{,\alpha}}{\varphi^2}\right) + 16\pi L_{\text{mat}}.$$

- Varying over $g_{\alpha\beta}$ and φ, we get the following equations:

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = \frac{8\pi}{\varphi}T_{\alpha\beta} + \frac{\omega}{\varphi^2}\left(\varphi_{,\alpha}\varphi_{,\beta} - \frac{1}{2}g_{\alpha\beta}\varphi_{,\gamma}\varphi^{,\gamma}\right) +$$

$$\frac{1}{\varphi}(\varphi_{;\alpha\beta} - g_{\alpha\beta}\Box\varphi);$$

$$\Box\varphi = \varphi^{;\alpha}_{;\alpha} = \frac{8\pi}{3+2\omega}T, \text{ where } T \stackrel{\text{def}}{=} T^{\alpha}_{\alpha}.$$

6. At First Glance, the Brans-Dicke Theory is T-Symmetric

- At first glance, the Brans-Dicke Theory (BDT) is similar to Einstein's General Relativity:
  - similar to General Relativity, the Brans-Dicke Theory is described by 2nd order PDEs, and
  - the BDT equations remain invariant if we reserve the order of time t, i.e., change t to $-t$.
- In general, in a second-order theory, if on some Cauchy surface (e.g., for some moment of time t_0),
  - we know the values of $g_{\alpha\beta}$, φ, and their first time derivatives $\dot{g}_{\alpha\beta}$ and $\dot{\varphi}$,
  - then we can uniquely determine the second time derivatives $\ddot{g}_{\alpha\beta}$ and $\ddot{\varphi}$,
  - and thus (at least locally) integrate the corresponding equations.

7. Main Result: Cauchy Problem for Brand-Dicker Theory (BDT) Leads to T-Asymmetry

- We show that if on some Cauchy surface,
  - we know the values of the gravity tensor $g_{\alpha\beta}$, its first time derivative $\dot{g}_{\alpha\beta}$, and the field φ,
  - then we can determine $\dot{\varphi}$ from a quadratic equation.
- A quadratic equation, in general, has *two* solutions.
- This means that in principle, for each initial condition, we can have *two* different dynamics.
- In physical terms, our result means that BDT, in effect, consists of *two* T-asymmetric theories.
- The transformation $t \to -t$ transforms each of these two theories into another one.
- This T-asymmetry may explain the observed time asymmetry of physical phenomena.

8. Let Us Use Gaussian Normal Coordinates

- When $g_{00} = 1$ and $g_{0i} = 0$ for $i = 1, 2, 3$, BDT equations take the form:

$$-\frac{1}{2}\dot{\varkappa}^i_i - \frac{1}{4}\varkappa^i_j \varkappa^j_i = \frac{8\pi}{\varphi}\left(T_{00} - \frac{1+\omega}{3+2\omega}T\right) + \omega\frac{(\dot{\varphi})^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi};$$

$$\frac{1}{2}\varkappa^j_{i;j} - \frac{1}{2}\varkappa^j_{j;i} = \frac{8\pi}{\varphi}T_{0i} + \omega\frac{\dot{\varphi}\varphi_{,i}}{\varphi^2} + \frac{\dot{\varphi}_{,i}}{\varphi};$$

$$P_{ij} - \frac{1}{2}\dot{\varkappa}_{ij} - \frac{1}{4}(\varkappa_{ij}\varkappa^k_k - 2\varkappa^k_i\varkappa_{kj}) =$$

$$\frac{8\pi}{\varphi}\left(T_{ij} + \frac{1+\omega}{3+2\omega}T\gamma_{ij}\right) + \omega\frac{\varphi_{,i}\varphi_{,j}}{\varphi^2} + \frac{\varphi_{;ij} - \varkappa_{ij}\dot{\varphi}}{\varphi};$$

$$\ddot{\varphi} - \Delta\varphi = \frac{8\pi}{3+2\omega}T.$$

- Here, $\gamma_{ij} \stackrel{\text{def}}{=} -g_{ij}$, $\varkappa_{ij} \stackrel{\text{def}}{=} -\dot{\gamma}_{ij}$, P_{ij} is the 3-D curvature tensor, and all tensor operations are w.r.t. γ_{ij}.

9. Proof: Idea

- From $\ddot{\varphi} - \Delta\varphi = \dfrac{8\pi}{3+2\omega}T$, we can explicitly express $\ddot{\varphi}$ in terms of γ_{ij}, $\dot{\gamma}_{ij}$, and φ.
- From the equation below, we can explicitly express $\dot{\varkappa}_{ij}$ (and, thus, $\dot{\varkappa}^i_i$) in terms of γ_{ij}, $\dot{\gamma}_{ij}$, φ, and $\dot{\varphi}$:

$$P_{ij} - \frac{1}{2}\dot{\varkappa}_{ij} - \frac{1}{4}(\varkappa_{ij}\varkappa^k_k - 2\varkappa^k_i\varkappa_{kj}) =$$

$$\frac{8\pi}{\varphi}\left(T_{ij} + \frac{1+\omega}{3+2\omega}T\gamma_{ij}\right) + \omega\frac{\varphi_{,i}\varphi_{,j}}{\varphi^2} + \frac{\varphi_{;ij} - \varkappa_{ij}\dot{\varphi}}{\varphi}.$$

- The resulting dependence of $\dot{\varkappa}^i_i$ on $\dot{\varphi}$ is linear.
- Substituting these expression for $\dot{\varkappa}^i_i$ and $\ddot{\varphi}$ into the equation below, we get a quadratic equation for $\dot{\varphi}$:

$$-\frac{1}{2}\dot{\varkappa}^i_i - \frac{1}{4}\varkappa^i_j\varkappa^j_i = \frac{8\pi}{\varphi}\left(T_{00} - \frac{1+\omega}{3+2\omega}T\right) + \omega\frac{(\dot{\varphi})^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi}.$$

10. The Same T-asymmetry Holds for More General Scalar-Tensor Theories of Gravitation

- Physicists also consider generalizations of Brans-Dicke theory, with the Lagrangian
$$L = \varphi\left(R - \omega \cdot \frac{\varphi_{,\alpha}\varphi^{,\alpha}}{\varphi^2} - V(\varphi)\right) + 16\pi L_{\text{mat}}.$$
- Here, the variational equations take the form
$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = \frac{8\pi}{\varphi}T_{\alpha\beta} + \frac{\omega}{\varphi^2}\left(\varphi_{,\alpha}\varphi_{,\beta} - \frac{1}{2}g_{\alpha\beta}\varphi_{,\gamma}\varphi^{,\gamma}\right) +$$
$$\frac{1}{\varphi}(\varphi_{;\alpha\beta} - g_{\alpha\beta}\Box\varphi) - \frac{1}{2}\frac{V(\varphi)}{\varphi}g_{\alpha\beta};$$
$$\Box\varphi = \varphi^{;\alpha}_{;\alpha} = \frac{8\pi}{3+2\omega}T - \frac{1}{3+2\omega}\left(V - \varphi\frac{dV}{d\varphi}\right).$$
- The two additional terms depend only on φ.
- So, $\dot{\varphi}$ can still be (almost) uniquely determined by the initial values of φ, g_{ij}, and $\dot{g}_{ij}$.

11. A Similar Phenomenon Also Holds for More Traditional Scalar-Tensor Theories

- In more traditional theories,

$$L = \frac{1}{G} R + L_{\text{scalar}}(\varphi, \varphi_\alpha \varphi^{,\alpha}) + 16\pi L_{\text{mat}}.$$

- For these theories, it is even easier to prove that we can reconstruct $\dot\varphi$ from φ, g_{ij} and $\dot g_{ij}$.
- Indeed, the RHS of the Einstein equations $R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = \ldots$ depends only on the first derivatives $\dot\varphi$ and $\varphi_{,i}$ of φ.
- In particular, the right-hand side of the equation corresponding to R_{0i} contains only $\dot\varphi$.
- This right-hand side can thus be used to explicitly express $\dot\varphi$ in terms of φ, g_{ij} and $\dot g_{ij}$.

12. Conclusion

- The time-symmetric Brans-Dicke Theory of gravitation (BDT), in effect, consists of two different theories.
- Each solution of BDT is a solution of one of these two theories.
- In particular, our Universe satisfies one of the corresponding two systems of partial differential equations.
- The transformation $t \to -t$ transforms each of these two theories into another one.
- However, none of these two theories is time-symmetric.
- So, in the presence of the additional scalar field, physical equations are *not* time symmetric.
- This may explain the observed time asymmetry of physical phenomena.

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