Equations Without Equations: Challenges on a Way to a More Adequate Formalization of Reasoning in Physics

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1. Need to Formalize Reasoning in Physics

- *Fact:* in medicine, geophysics, etc., expert systems use automated expert reasoning to help the users.
- *Hope:* similar systems may be helpful in general theoretical physics as well.
- *What is needed:* describe physicists' reasoning in precise terms.
- *Reason:* formalize this reasoning inside an automated computer system.
- *Formalized part of physicists' reasoning:* theories are formulated in terms of PDEs (or ODEs) $\frac{dx}{dt} = F(x)$.
- *Meaning:* these equations describe how the corresponding fields (or quantities) x change with time t.

2. Mathematician's View of Physics and Its Limitations

- *Mathematician's view:* we know the initial conditions $x(t_0)$ at some moment of time t_0.
- We solve the corresponding Cauchy problem and find the values $x(t)$ for all t.
- *Limitation:* not all solutions to the equation $\dfrac{dx}{dt} = F(x)$ are physically meaningful.
- *Example 1:* when a cup breaks into pieces, the corresponding trajectories of molecules make physical sense.
- *Example 2:* when we reverse all the velocities, we get pieces assembling themselves into a cup.
- *Fact:* this is physically impossible.
- *Fact:* the reverse process satisfies all the original (T-invariant) equations.

3. Physicists' Explanation

- *Reminder:* not all solutions to the physical equation are physically meaningful.
- *Explanation:* the "time-reversed" solution is non-physical because its initial conditions are "degenerate".
- *Clarification:* once we modify the initial conditions even slightly, the pieces will no longer get together.
- *Conclusion:* not only the equations must be satisfied, but also the initial conditions must be "non-degenerate".
- *Two challenges* in formalizing this idea:
  - how to formalize "non-degenerate";
  - the separation between equations and initial conditions depends on the way equations are presented.
- *First challenge:* can be resolved by using Kolmogorov complexity and randomness.

4. First Example: Schrödinger's Equation

- *Example:* Schrödinger's equation

$$i\hbar \cdot \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \cdot \nabla^2 \Psi + V(\vec{r}) \cdot \Psi.$$

- *In this representation:* the potential V is a part of the equation, and $\Psi(\vec{r}, t_0)$ are initial conditions.

- *Transformation:*
  - we represent $V(\vec{r})$ as a function of Ψ and its derivatives,
  - differentiate the right-hand side by time, and
  - equate the derivative w.r.t. time to 0.

- *Result:*

$$\frac{\partial}{\partial t}\left(\frac{i\hbar}{\Psi} \cdot \frac{\partial \Psi}{\partial t} + \frac{\hbar^2}{2m} \cdot \frac{\nabla^2 \Psi}{\Psi}\right) = 0.$$

5. First Example (cont-d)

- *Reminder:*

$$\frac{\partial}{\partial t}\left(\frac{i\hbar}{\Psi}\cdot\frac{\partial\Psi}{\partial t}+\frac{\hbar^2}{2m}\cdot\frac{\nabla^2\Psi}{\Psi}\right)=0.$$

- *Mathematically:* the new equation (2nd order in time) is equivalent to the Schrödinger's equation:
  - every solution of the Schrödinger's equation for any $V(\vec r)$ satisfies this new equation, and
  - every solution of the new equation satisfies Schödinger's equation for some $V(\vec r)$.
- *Observation:* in the new equation, initial conditions, in effect, include $V(\vec r)$.
- *Conclusion:* "non-degeneracy" ("randomness") condition must now include $V(\vec r)$ as well.

6. Second Example: General Scalar Field

- *Example:* consider a scalar field φ with a generic Lagrange function $L(\varphi, a)$, with $a \stackrel{\text{def}}{=} \varphi_{,i}\varphi^{,i}$.
- *Traditional formulation:* every Lagrangian is possible, but initial conditions $\varphi(x, t_0)$ must be non-degenerate.
- *Euler equations:* $\dfrac{\partial L}{\partial \varphi} - \partial_i \dfrac{\partial L}{\partial \varphi_{,i}} = L_{,\varphi} - \partial_i (2L_{,a} \cdot \varphi_{,i}) = 0$:

$$L_{,\varphi} - 2L_{,a} \cdot \Box\, \varphi - 2L_{,a\varphi} \cdot (\varphi_{,i}\varphi^{,i}) - 4L_{,aa} \cdot \varphi_{,ij}\varphi^{,i}\varphi^{,j} = 0.$$

- In general, on a 3-D Cauchy surface $t = t_0$, we can find points with arbitrary combination of $(\varphi, \varphi_{,i}\varphi^{,i}, \Box\, \varphi)$.
- Thus, by observing the evolution, we can find $\varphi_{,ij}\varphi^{,i}\varphi^{,j}$ for all possible triples $(\varphi, \varphi_{,i}\varphi^{,i}, \Box\, \varphi)$.
- So, we can predict future evolution – w/o knowing L.

7. Scalar Field: Discussion and Conclusions

- *Observation:* the new "equation" does not contain L at all.
- *Fact:* a field φ satisfies the new equation $\Leftrightarrow$ it satisfies the Euler-Lagrange equations for *some* L.
- *Observation:*
  - similarly to Wheeler's cosmological "mass without mass" and "charge without charge",
  - we now have "equations without equations".
- *Conclusion:* when formalizing physical equations:
  - we must not only describe them in *a* mathematical form,
  - we must also select *one* of the mathematically equivalent forms.

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