

# Quantum Social Science

(A review of a recent book by  
E. Haven and A. Khrennikov)

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## 1. Outline

- Decision theory – how to describe preferences if people behave rationally.
- In practice, people sometimes deviate from rational behavior.
- In some cases, quantum-type models are helpful in explaining actual human behavior.
- The authors use this to explain fluctuations of the stock market prices.
- They also speculate on why humans use quantum-type reasoning.

## 2. Decision Making: General Need and Traditional Approach

- To make a decision, we must:
  - find out the user's preference, and
  - help the user select an alternative which is the best
    - according to these preferences.
- Traditional approach is based on an assumption that for each two alternatives  $A'$  and  $A''$ , a user can tell:
  - whether the first alternative is better for him/her; we will denote this by  $A' < A''$ ;
  - or the second alternative is better; we will denote this by  $A' < A''$ ;
  - or the two given alternatives are of equal value to the user; we will denote this by  $A' = A''$ .

### 3. The Notion of Utility

- Under the above assumption, we can form a natural numerical scale for describing preferences.
- Let us select a very bad alternative  $A_0$  and a very good alternative  $A_1$ .
- Then, most other alternatives are better than  $A_0$  but worse than  $A_1$ .
- For every prob.  $p \in [0, 1]$ , we can form a lottery  $L(p)$  in which we get  $A_1$  w/prob.  $p$  and  $A_0$  w/prob.  $1 - p$ .
- When  $p = 0$ , this lottery simply coincides with the alternative  $A_0$ :  $L(0) = A_0$ .
- The larger the probability  $p$  of the positive outcome increases, the better the result:

$$p' < p'' \text{ implies } L(p') < L(p'').$$

## 4. The Notion of Utility (cont-d)

- Finally, for  $p = 1$ , the lottery coincides with the alternative  $A_1$ :  $L(1) = A_1$ .
- Thus, we have a continuous scale of alternatives  $L(p)$  that monotonically goes from  $L(0) = A_0$  to  $L(1) = A_1$ .
- Due to monotonicity, when  $p$  increases, we first have  $L(p) < A$ , then we have  $L(p) > A$ .
- The threshold value is called the *utility* of the alternative  $A$ :

$$u(A) \stackrel{\text{def}}{=} \sup\{p : L(p) < A\} = \inf\{p : L(p) > A\}.$$

- Then, for every  $\varepsilon > 0$ , we have

$$L(u(A) - \varepsilon) < A < L(u(A) + \varepsilon).$$

- We will describe such (almost) equivalence by  $\equiv$ , i.e., we will write that  $A \equiv L(u(A))$ .

## 5. Fast Iterative Process for Determining $u(A)$

- *Initially:* we know the values  $\underline{u} = 0$  and  $\bar{u} = 1$  such that  $A \equiv L(u(A))$  for some  $u(A) \in [\underline{u}, \bar{u}]$ .
- *What we do:* we compute the midpoint  $u_{\text{mid}}$  of the interval  $[\underline{u}, \bar{u}]$  and compare  $A$  with  $L(u_{\text{mid}})$ .
- *Possibilities:*  $A \leq L(u_{\text{mid}})$  and  $L(u_{\text{mid}}) \leq A$ .
- *Case 1:* if  $A \leq L(u_{\text{mid}})$ , then  $u(A) \leq u_{\text{mid}}$ , so

$$u \in [\underline{u}, u_{\text{mid}}].$$

- *Case 2:* if  $L(u_{\text{mid}}) \leq A$ , then  $u_{\text{mid}} \leq u(A)$ , so

$$u \in [u_{\text{mid}}, \bar{u}].$$

- After each iteration, we decrease the width of the interval  $[\underline{u}, \bar{u}]$  by half.
- After  $k$  iterations, we get an interval of width  $2^{-k}$  which contains  $u(A)$  – i.e., we get  $u(A)$  w/accuracy  $2^{-k}$ .

## 6. How to Make a Decision Based on Utility Values

- Suppose that we have found the utilities  $u(A')$ ,  $u(A'')$ ,  $\dots$ , of the alternatives  $A'$ ,  $A''$ ,  $\dots$
- Which of these alternatives should we choose?
- By definition of utility, we have:
  - $A \equiv L(u(A))$  for every alternative  $A$ , and
  - $L(p') < L(p'')$  if and only if  $p' < p''$ .
- We can thus conclude that  $A'$  is preferable to  $A''$  if and only if  $u(A') > u(A'')$ .
- In other words, we should always select an alternative with the largest possible value of utility.

## 7. How to Estimate Utility of an Action

- For each action, we usually know possible outcomes  $S_1, \dots, S_n$ .
- We can often estimate the prob.  $p_1, \dots, p_n$  of these outcomes.
- By definition of utility, each situation  $S_i$  is equiv. to a lottery  $L(u(S_i))$  in which we get:
  - $A_1$  with probability  $u(S_i)$  and
  - $A_0$  with the remaining probability  $1 - u(S_i)$ .
- Thus, the action is equivalent to a complex lottery in which:
  - first, we select one of the situations  $S_i$  with probability  $p_i$ :  $P(S_i) = p_i$ ;
  - then, depending on  $S_i$ , we get  $A_1$  with probability  $P(A_1 | S_i) = u(S_i)$  and  $A_0$  w/probability  $1 - u(S_i)$ .



## 8. How to Estimate Utility of an Action (cont-d)

- *Reminder:*

- first, we select one of the situations  $S_i$  with probability  $p_i$ :  $P(S_i) = p_i$ ;
- then, depending on  $S_i$ , we get  $A_1$  with probability  $P(A_1 | S_i) = u(S_i)$  and  $A_0$  w/probability  $1 - u(S_i)$ .

- The prob. of getting  $A_1$  in this complex lottery is:

$$P(A_1) = \sum_{i=1}^n P(A_1 | S_i) \cdot P(S_i) = \sum_{i=1}^n u(S_i) \cdot p_i.$$

- In the complex lottery, we get:

- $A_1$  with prob.  $u = \sum_{i=1}^n p_i \cdot u(S_i)$ , and
- $A_0$  w/prob.  $1 - u$ .

- So, we should select the action with the largest value of expected utility  $u = \sum p_i \cdot u(S_i)$ .

## 9. Non-Uniqueness of Utility

- The above definition of utility  $u$  depends on  $A_0, A_1$ .
- What if we use different alternatives  $A'_0$  and  $A'_1$ ?
- Every  $A$  is equivalent to a lottery  $L(u(A))$  in which we get  $A_1$  w/prob.  $u(A)$  and  $A_0$  w/prob.  $1 - u(A)$ .
- For simplicity, let us assume that  $A'_0 < A_0 < A_1 < A'_1$ .
- Then,  $A_0 \equiv L'(u'(A_0))$  and  $A_1 \equiv L'(u'(A_1))$ .
- So,  $A$  is equivalent to a complex lottery in which:
  - 1) we select  $A_1$  w/prob.  $u(A)$  and  $A_0$  w/prob.  $1 - u(A)$ ;
  - 2) depending on  $A_i$ , we get  $A'_1$  w/prob.  $u'(A_i)$  and  $A'_0$  w/prob.  $1 - u'(A_i)$ .
- In this complex lottery, we get  $A'_1$  with probability  $u'(A) = u(A) \cdot (u'(A_1) - u'(A_0)) + u'(A_0)$ .
- So, in general, utility is defined modulo an (increasing) linear transformation  $u' = a \cdot u + b$ , with  $a > 0$ .

## 10. Subjective Probabilities

- In practice, we often do not know the probabilities  $p_i$  of different outcomes.
- For each event  $E$ , a natural way to estimate its subjective probability is to fix a prize (e.g., \$1) and compare:
  - the lottery  $\ell_E$  in which we get the fixed prize if the event  $E$  occurs and 0 if it does not occur, with
  - a lottery  $\ell(p)$  in which we get the same amount with probability  $p$ .
- Here, similarly to the utility case, we get a value  $ps(E)$  for which, for every  $\varepsilon > 0$ :

$$\ell(ps(E) - \varepsilon) < \ell_E < \ell(ps(E) + \varepsilon).$$

- Then, the utility of an action with possible outcomes  $S_1, \dots, S_n$  is equal to  $u = \sum_{i=1}^n ps(E_i) \cdot u(S_i)$ .

## 11. Example of Seemingly Irrational Behavior

- An urn contains red ( $R$ ), green ( $G$ ), and blue ( $B$ ) balls.
- We know that exactly  $1/3$  of balls are red:  $p(R) = 1/3$ .
- We do not know  $p(G)$ .

- 1st experiment: choose between two alternatives:

$R$ : \$1 if a randomly chosen ball is red, \$0 otherwise;

$G$ : \$1 if a random ball is green, \$0 otherwise.

- Result: most people select alternative  $R$ :  $G < R$ .
- In terms of utility: we can take  $u(\$0) = 0$ ,  $u(\$1) = 1$ .
- In this case,  $u(R) = (1/3) \cdot u(\$1) + (2/3) \cdot u(\$0) = 1/3$  and  $u(G) = ps(G) \cdot u(\$1) + (1 - ps(G)) \cdot u(\$0) = ps(G)$ .
- Since  $G < R$ , this means that  $ps(G) < 1/3$ .

## 12. Seemingly Irrational Behavior (cont-d)

- *Second experiment:* a person is asked to choose between two alternatives:

*GB:* you get \$1 if a randomly picked ball is either green or blue and \$0 otherwise;

*RB:* you get \$1 if a randomly picked ball is either red or blue and \$0 otherwise.

- *Result:* most people select alternative *GB*:  $RB < GB$ .
- *In terms of utility:*

$$u(GB) = (2/3) \cdot u(\$1) + (1/3) \cdot u(\$0) = 2/3;$$

$$u(RB) = (1 - ps(G)) \cdot u(\$1) + ps(G) \cdot u(\$0) = 1 - ps(G).$$

- Since  $RB < GB$ , this means that for most people,  $1 - ps(G) < 2/3$  and thus,  $ps(G) > 1/3$ .
- *Contradiction:* this conclusion is inconsistent with the previous conclusion  $ps(G) < 1/3$ .

## 13. Relation to Quantum Physics

- Here,  $ps(G) < 1/3$ ,  $ps(B) < 1/3$ , but  $ps(G \vee B) = 2/3$ .
- In other words, here,  $ps(G \vee B) \neq ps(G) + ps(B)$ .
- Such situations are typical for *quantum processes*.
- *Example*: a photon source is separated from the sensors by a wall with two doors.
- If we open one door or both doors, some photons pass to the sensors.
- We can open the first door and count how many photos passed to the sensors.
- Let  $p_1$  be the resulting probability.
- We can open the second door and count how many photos passed to the sensors.
- Let  $p_2$  be the resulting probability.

## 14. Relation to Quantum Physics (cont-d)

- If both doors are open, then a photon can pass:
  - either through the 1st door
  - or through the 2nd door
  - but not through both doors.
- So, we expect  $p_{12} = p_1 + p_2$ .
- In practice,  $p_{12} \neq p_1 + p_2$ .
- *Quantum description:* we have complex *amplitudes*  $\Psi_1$  and  $\Psi_2$  such that  $p_1 = |\Psi_1|^2$  and  $p_2 = |\Psi_2|^2$ .
- Here,  $p_{12} = |\Psi_{12}|^2$ , where  $\Psi_{12} = \Psi_1 + \Psi_2$ .
- In general,  $|\Psi_1 + \Psi_2|^2 \neq |\Psi_1|^2 + |\Psi_2|^2$ , so  $p_{12} \neq p_1 + p_2$ .
- *Interesting:* kids going to candy boxes behave similarly to photons.

## 15. From Individual to Collective Behavior

- Psychologists have performed many experiments showing such irrational behavior.
- Several models have been proposed explaining such behavior.
- These models, however, are far from perfect when describing individual decision making.
- At first glance, the problem of describing such irrational behavior,
  - while important in psychology,
  - is not of large practical interest.
- The authors notice, however, that:
  - a combination of such irrational behaviors
  - leads, e.g., to seemingly irrational fluctuations of the stock market.



## 16. Collective Behavior (cont-d)

- In contrast to highly irregular individual behavior, such group behavior is much more regular.
- It can be reasonably well described by known mathematical models.
- For example, we can use quantum-type stochastic differential equations.
- In the first approximation, the resulting equations are similar to Schroedinger's equations of quantum physics.
- A detailed analysis shows that the stock market dynamics is somewhat different from quantum dynamics.

## 17. Alternative (Non-Quantum) Explanation

- Previously, we assumed that a user can always decide which of the two alternatives  $A'$  and  $A''$  is better:
  - either  $A' < A''$ ,
  - or  $A'' < A'$ ,
  - or  $A' \equiv A''$ .
- In practice, a user is sometimes unable to meaningfully decide between the two alternatives; denoted  $A' \parallel A''$ .
- In mathematical terms, this means that the preference relation:
  - is no longer a *total* (linear) order,
  - it can be a *partial* order.

## 18. From Utility to Interval-Valued Utility

- Similarly to the traditional decision making approach:
  - we select two alternatives  $A_0 < A_1$  and
  - we compare each alternative  $A$  which is better than  $A_0$  and worse than  $A_1$  with lotteries  $L(p)$ .

- Since preference is a *partial* order, in general:

$$\underline{u}(A) \stackrel{\text{def}}{=} \sup\{p : L(p) < A\} < \bar{u}(A) \stackrel{\text{def}}{=} \inf\{p : L(p) > A\}.$$

- For each alternative  $A$ , instead of a single value  $u(A)$  of the utility, we now have an *interval*  $[\underline{u}(A), \bar{u}(A)]$  s.t.:
  - if  $p < \underline{u}(A)$ , then  $L(p) < A$ ;
  - if  $p > \bar{u}(A)$ , then  $A < L(p)$ ; and
  - if  $\underline{u}(A) < p < \bar{u}(A)$ , then  $A \parallel L(p)$ .
- We will call this interval the *utility* of the alternative  $A$ .

## 19. Interval-Valued Utilities and Interval-Valued Subjective Probabilities

- To feasibly elicit the values  $\underline{u}(A)$  and  $\bar{u}(A)$ , we:
  - 1) starting w/ $[\underline{u}, \bar{u}] = [0, 1]$ , bisect an interval s.t.  
 $L(\underline{u}) < A < L(\bar{u})$  until we find  $u_0$  s.t.  $A \parallel L(u_0)$ ;
  - 2) by bisecting an interval  $[\underline{u}, u_0]$  for which  
 $L(\underline{u}) < A \parallel L(u_0)$ , we find  $\underline{u}(A)$ ;
  - 3) by bisecting an interval  $[u_0, \bar{u}]$  for which  
 $L(u_0) \parallel A < L(\bar{u})$ , we find  $\bar{u}(A)$ .
- Similarly, when we estimate the probability of an event  $E$ :
  - we no longer get a single value  $ps(E)$ ;
  - we get an *interval*  $[\underline{ps}(E), \bar{ps}(E)]$  of possible values of probability.
- By using bisection, we can feasibly elicit the values  $\underline{ps}(E)$  and  $\bar{ps}(E)$ .

## 20. Decision Making Under Interval Uncertainty

- *Situation*: for each possible decision  $d$ , we know the interval  $[\underline{u}(d), \bar{u}(d)]$  of possible values of utility.
- *Questions*: which decision shall we select?
- *Natural idea*: select all decisions  $d_0$  that *may* be optimal, i.e., which are optimal for some function

$$u(d) \in [\underline{u}(d), \bar{u}(d)].$$

- *Problem*: checking all possible functions is not feasible.
- *Solution*: the above condition is equivalent to an easier-to-check one:

$$\bar{u}(d_0) \geq \max_d \underline{u}(d).$$

- *Interval computations* can help in describing the range of all such  $d_0$ .
- *Remaining problem*: in practice, we would like to select *one* decision; which one should be select?

## 21. Decisions under Interval Uncertainty: Hurwicz Optimism-Pessimism Criterion

- *Reminder:* we need to assign, to each interval  $[\underline{u}, \bar{u}]$ , a utility value  $u(\underline{u}, \bar{u}) \in [\underline{u}, \bar{u}]$ .
- *History:* this problem was first handled in 1951, by the future Nobelist Leonid Hurwicz.
- *Notation:* let us denote  $\alpha_H \stackrel{\text{def}}{=} u(0, 1)$ .
- *Reminder:* utility is determined modulo a linear transformation  $u' = a \cdot u + b$ .
- *Reasonable to require:* the equivalent utility does not change with re-scaling: for  $a > 0$  and  $b$ ,

$$u(a \cdot u^- + b, a \cdot u^+ + b) = a \cdot u(u^-, u^+) + b.$$

- For  $u^- = 0$ ,  $u^+ = 1$ ,  $a = \bar{u} - \underline{u}$ , and  $b = \underline{u}$ , we get

$$u(\underline{u}, \bar{u}) = \alpha_H \cdot (\bar{u} - \underline{u}) + \underline{u} = \alpha_H \cdot \bar{u} + (1 - \alpha_H) \cdot \underline{u}.$$

## 22. Hurwicz Optimism-Pessimism Criterion (cont)

- The expression  $\alpha_H \cdot \bar{u} + (1 - \alpha_H) \cdot \underline{u}$  is called *optimism-pessimism criterion*, because:
  - when  $\alpha_H = 1$ , we make a decision based on the most optimistic possible values  $u = \bar{u}$ ;
  - when  $\alpha_H = 0$ , we make a decision based on the most pessimistic possible values  $u = \underline{u}$ ;
  - for intermediate values  $\alpha_H \in (0, 1)$ , we take a weighted average of the optimistic and pessimistic values.
- According to this criterion:
  - if we have several alternatives  $A', \dots$ , with interval-valued utilities  $[\underline{u}(A'), \bar{u}(A')]$ ,  $\dots$ ,
  - we recommend an alternative  $A$  that maximizes

$$\alpha_H \cdot \bar{u}(A) + (1 - \alpha_H) \cdot \underline{u}(A).$$

## 23. Alternative Explanation of the Seemingly Irrational Human Behavior

- *Situation:* an urn contains red ( $R$ ), green ( $G$ ), and blue ( $B$ ) balls;  $p(R) = 1/3$ .
- *We do not know:* the proportion of green and blue balls, so we only know that  $p(G) \in [0, 2/3]$ .
- *1st experiment:* choose between two alternatives:  
     $R$ : \$1 if a random ball is red, \$0 otherwise;  
     $G$ : \$1 if a random ball is green, \$0 otherwise.
- In this case,  $u(R) = (1/3) \cdot u(\$1) + (2/3) \cdot u(\$0) = 1/3$ .
- Here,  $u(G) = p(G) \cdot u(\$1) + (1 - p(G)) \cdot u(\$0) = p(G)$ .
- So, possible values of  $u(G)$  form an interval  $[0, 2/3]$ .
- This is equivalent to  $\alpha_H \cdot (2/3) + (1 - \alpha_H) \cdot 0 = (2/3) \cdot \alpha_H$ .
- Since  $G < R$ , we have  $(2/3) \cdot \alpha_H < 1/3$ , so  $\alpha_H < 1/2$ .



## 24. Alternative Explanation (cont-d)

- *2nd experiment*: choose between two alternatives:

*GB*: \$1 if a random ball is green or blue, \$0 otherwise;

*RB*: \$1 if a random ball is red or blue, \$0 otherwise.

- *Result*: most people select alternative *GB*:  $RB < RB$ .
- Here,  $u(GB) = (2/3) \cdot u(\$1) + (2/3) \cdot u(\$0) = 2/3$ .
- In this case,  $p(RB) = 1 - p(G)$ , so

$$u(RB) = (1 - p(G)) \cdot u(\$1) + p(G) \cdot u(\$0) = 1 - p(G).$$

- Since  $ps(G) \in [0, 2/3]$ , we have  $u(RB) \in [1/3, 1]$ , which is equiv. to  $\alpha_H \cdot 1 + (1 - \alpha_H) \cdot (1/3) = (2/3) \cdot \alpha_H + 1/3$ .
- $RB < GB$  means  $(2/3) \cdot \alpha_H + 1/3 < 2/3$ , i.e.,  $\alpha_H < 1/2$ .
- This is the same restriction on  $\alpha_H$  that we obtained from the 1st experiment.

## 25. Why Humans Use Quantum-Type Reasoning?

- The authors also speculate on why humans use quantum-type reasoning.
- One possible reason:
  - many real-life processes are quantum, and
  - we want to simulate them.
- Another possible reason:
  - our brain, as a product of billion years of evolution, implements the best possible algorithms, and
  - quantum algorithms are known to be very efficient.

## 26. Why Quantum Computing

- The speed of all processes is limited by the speed of light  $c$ .
- To send a signal across a 30 cm laptop, we need at least 1 ns; this corresponds to only 1 Gflop.
- If we want to make computers faster, we need to make processing elements smaller.
- Already, each processing cell consists of a few dozen molecules.
- If we decrease the size further, we get to the level of individual atoms and molecules.
- On this level, physics is different, it is quantum physics.
- One of the properties of quantum physics is its probabilistic nature (example: radioactive decay).

## 27. Why Quantum Computing (cont-d)

- At first glance, this interferes with our desire to make reproducible computations.
- However, scientists learned how to make lemonade out of this lemon.
- First main discovery: Grover's quantum search algorithm.
- To search for an object in an unsorted array of  $n$  elements, we need, in the worst case, at least  $n$  steps.
- Reason: if we use fewer steps, we do not cover all the elements, and thus, we may miss the desired object.
- In quantum physics, we can find an element in  $\sqrt{n}$  steps.
- For a Terabyte database, we get a million times speedup.
- Main idea: we can use superposition of different searches.

## 28. Why Quantum Computing (cont-d)

- Another discovery: Shor's cracking RSA coding.
- The RSA algorithm is behind most secure transactions.
- A person selects two large prime numbers  $P_1$  and  $P_2$ , and advertises their product  $n = P_1 \cdot P_2$ .
- By using this open code  $n$ , anyone can encode their message.
- To decode this message, one needs to know the factors  $P_1$  and  $P_2$ .
- Factoring a large integer is known to be a computationally difficult problem.
- It turns out that with quantum computers, we can factor fast and thus, read all encrypted messages.
- The situation is not so bad: there is also a quantum encryption which cannot be easily cracked.

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