Metrological Challenges of Practical Computer-Enhanced Measurements

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1. Formulation of the problems and overview of the thesis

- As technology progresses, sensors and computers become cheaper.
- As a result, we can afford more measurements, and we can afford to process them faster and better.
- However, this progress also brings challenges.
- In this thesis, we list these challenges in Chapter 1.
- In the following chapters, we describe examples of these challenges, and provide possible solutions.
- The first challenge is related to the fact that:
  - the existing metrological recommendations are mostly based on the previous practice,
  - when we could only afford to have a small number of measurements.
- We show that if we naively use these recommendations, we may encounter serious problems.

2. Overview of the Thesis (cont-d)

- In Chapter 2, we illustrate this problem on the example of the design of the Thermonuclear Research Center.
- We also explain how this challenge can be resolved.
- Another challenge is related to the fact that:
  - in the past, when there were few affordable measuring instruments and we could only afford a few measurements,
  - there were not that many options.
- So, we could select one of these options "by hand".
- Nowadays, there is a potential to perform a large number of measurements.
- Many different measuring instruments are available.
- So, the number of possible measurement options has become large.
- It is therefore important to develop methods for optimal planning.

3. Overview of the Thesis (cont-d)

- In Chapter 3, we propose such a method for an important case of distributed measurements.
- The third challenge is related to the fact that:
  - with the possibility to perform numerous measurements and process their results,
  - we often encounter situations when different pairs of measurement errors we have different types of information.
- Some are known to be independent.
- For others, we do not have such information.
- In Chapter 4, we develop algorithms for dealing with such situations.
- The final – fourth – challenge is how to extract useful information from all these measurement results.
- In Chapter 5, we handle an important particular case of this challenge: finding faults in a smart electric grid.

4. Overview of the Thesis (cont-d)

- In this presentation, we provide:
  - a brief description of the first and third challenges, and
  - a detailed description of second and fourth challenges.

5. FIRST CHALLENGE: a brief description

- Measurement errors are usually normally distributed.
- For a normal distribution, the probability to be outside 3σ interval is 0.1%.
- So, when we perform $\approx$40 measurements, with high probability all of them are within 3σ interval.
- Because of this, the current standards require that all measurement results are within this interval.
- However, now, sensors are cheap.
- We can (and often do) perform thousands of measurements.
- Inevitably, some of them will be outside the 3σ interval; so:
  - we use sensors which are as accurate as before, but
  - by the current standards, the measuring system is not approved.

6. First challenge: a brief description

- This is not just a hypothetical issue.
- This stopped the design of the International Thermonuclear Research Center.
- Our solution: update the standards, so that
  - if a measuring system was previously approved,
  - if should be approved now, when we perform more measurements.

7. SECOND CHALLENGE: motivations

- In many practical situations, we are interested in estimating the value x of a cumulative quantity; e.g, we want to estimate:
  - the overall amount of oil in a given area,
  - the overall amount of CO_2 emissions, etc.
- Measuring instruments usually measure quantities in a given location.
- Thus, they measure local values $x_1, \ldots, x_n$ that together form the desired value $x = x_1 + \ldots + x_n$.
- So, a natural way to produce an estimate $\widetilde{x}$ for x is:
  - to place measuring instruments at several locations,
  - to measure the values x_i in these locations, and
  - to add up the results $\widetilde{x}_1 + \ldots + \widetilde{x}_n$ of these measurement:

$$\widetilde{x} = \widetilde{x}_1 + \ldots + \widetilde{x}_n.$$

8. Need for optimal planning

- Usually, we want to reach a certain estimation accuracy.
- To achieve this accuracy, we need to plan how accurate the deployed measurement instruments should be.
- Use of accurate measuring instruments is often very expensive, while budgets are usually limited.
- It is therefore desirable to come up with the deployment plan that would achieve the desired overall accuracy within the minimal cost.
- This implies, in particular, that the resulting estimate should not be more accurate than needed.
- Indeed, this would mean that we could use less accurate (and thus, cheaper) measuring instruments.

9. Need for optimal planning (cont-d)

- In this talk, we provide:
  - a condition under which such optimal planning is possible, and
  - the corresponding optimal planning algorithm.
- The resulting condition explains why usually, measuring instruments are characterized by their absolute and relative accuracy.

10. How we can describe measurement accuracy

- Measurements are never absolutely accurate.
- The measurement result $\widetilde{x}_i$ is, in general, different from the actual (unknown) value x_i of the corresponding quantity.
- In other words, the difference $\Delta x_i \stackrel{\text{def}}{=} \widetilde{x}_i - x_i$ is, in general, different from 0.
- This difference is known as the *measurement error*.
- For each measuring instrument, we should know how large the measurement error can be.
- In precise terms, we need to know an upper bound Δ on the absolute value $|\Delta x_i|$ of the measurement error.
- This upper bound should be provided by the manufacturer of the measuring instrument.

11. How we can describe measurement accuracy (cont-d)

- Indeed, if no such upper bound is known, this means that:
  - whatever the reading of the measuring instrument,
  - the actual value can be as far away from this reading as possible.
- So we get no information whatsoever about the actual value – in this case, this is not a measuring instrument, it is a wild guess.
- Ideally:
  - in addition to knowing that the measurement error Δx_i is somewhere in the interval $[-\Delta, \Delta]$,
  - it is desirable to know how probable are different values from this interval,
  - i.e., what is the probability distribution on the measurement error.
- Sometimes, we know this probability distribution.
- However, in many practical situations, we don't know it, and the upper bound is all we know.

12. How we can describe measurement accuracy (cont-d)

- So, we will consider this upper bound as the measure of the instrument's accuracy.
- This upper bound Δ may depend on the measured value.
- For example, suppose that we are measuring current in the range from 1 mA to 1 A.
- Then, it is relatively easy to maintain accuracy of 0.1 mA when the actual current is 1 mA.
- This means measuring with one correct decimal digit.
- We can get values $0.813\ldots$, $0.825\ldots$
- However, since the measurement accuracy is 0.1, this means that these measurement results may correspond to the same actual value.
- In other words, whatever the measuring instrument shows, only one digit is meaningful and significant.

13. How we can describe measurement accuracy (cont-d)

- All the other digits may be caused by measurement errors.
- But can we maintain the same accuracy of 0.1 mA when we measure currents close to 1 A?
- This would mean that we need to distinguish between values 0.94651 A = 946.51 mA and 0.94637 A = 946.37 mA.
- Indeed, the difference between these two values is larger than 0.1 mA.
- This would mean that we require that in the measurement result, we should have not one, but four significant digits.
- This would require much more accurate measurements.
- Because of this, we will explicitly take into account that the accuracy Δ depends on the measured value: $\Delta = \Delta(x)$.
- Usually, small changes in x lead to only small changes in the accuracy.
- So, we can safely assume that the dependence $\Delta(x)$ is smooth.

14. What we want

- We want to estimate the desired cumulative value x with some accuracy δ.
- In other words, we want to make sure that the difference between our estimate $\widetilde{x}$ and the actual value x does not exceed δ: $|\widetilde{x} - x| \le \delta$.
- The cumulative value is estimated based on n measurement results.
- As we have mentioned, the accuracy that we can achieve in each measurement, in general, depends on the measured value.
- The larger the value of the measured quantity, the more difficult it is to maintain the corresponding accuracy.
- It is therefore reasonable to conclude that:
  - whatever measuring instruments we use to measure each value x_i,
  - it will be more difficult to estimate the larger cumulative value x with the same accuracy.

15. What we want (cont-d)

- Thus, it makes sense to require that the desired accuracy δ should also depend on the value that we want to estimate $\delta = \delta(x)$.
- The larger the value x, the larger the uncertainty $\delta(x)$ that we can achieve.
- So, our problem takes the following form:
- We want to be able to estimate the cumulative value x with given accuracy $\delta(x)$.
- In other words, we are given a function $\delta(x)$ and we want to estimate the cumulative value with this accuracy
- We want to find the measuring instruments:
  - that would guarantee this estimation accuracy, and
  - that would be optimal for this task, i.e., that would not provide better accuracy than needed.

16. Let us describe what we want in precise terms

- Let us analyze what estimation accuracy we can achieve if we use:
  - for each of n measurements,
  - the measuring instrument characterized by the accuracy $\Delta(x)$.
- Let $\widetilde{x}_i$ be the i-th measurement result.
- Then, the actual value x_i of the corresponding quantity is located somewhere on the interval $[\widetilde{x}_i - \Delta(x_i), \widetilde{x}_i + \Delta(x_i)]$.
- The smallest possible value is $\widetilde{x}_i - \Delta(x_i)$.
- The largest possible value is $\widetilde{x}_i + \Delta(x_i)$.
- When we add the measurement results, we get the estimate $\widetilde{x} = \widetilde{x}_1 + \ldots + \widetilde{x}_n$ for the desired quantity x.
- What are the possible values of this quantity?

17. Let us describe what we want in precise terms (cont-d)

- The sum $x = x_1 + \ldots + x_n$ attains its smallest value if all values x_i are the smallest, i.e., when

  $x = (\widetilde{x}_1 - \Delta(x_1)) + \ldots + (\widetilde{x}_n - \Delta(x_n)) = (\widetilde{x}_1 + \ldots + \widetilde{x}_n) - (\Delta(x_1) + \ldots + \Delta(x_n)).$

- In this case, $x = \widetilde{x} - (\Delta(x_1) + \ldots + \Delta(x_n))$.
- Similarly, the sum $x = x_1 + \ldots + x_n$ attains its largest value if all values x_i are the largest, i.e., when

  $x = (\widetilde{x}_1 + \Delta(x_1)) + \ldots + (\widetilde{x}_n + \Delta(x_n)) = (\widetilde{x}_1 + \ldots + \widetilde{x}_n) + (\Delta(x_1) + \ldots + \Delta(x_n)).$

- In this case, $x = \widetilde{x} + (\Delta(x_1) + \ldots + \Delta(x_n))$.
- Thus, all we can conclude about the value x is that this value belongs to the interval

  $$[\widetilde{x} - (\Delta(x_1) + \ldots + \Delta(x_n)), \widetilde{x} + (\Delta(x_1) + \ldots + \Delta(x_n))].$$

- This means that we get an estimate of x with the accuracy

  $$\Delta(x_1) + \ldots + \Delta(x_n).$$

18. Let us describe what we want in precise terms (cont-d)

- Our objective is to make sure that this is exactly the desired accuracy $\delta(x)$.
- In other words, we want to make sure that whenever $x = x_1 + x_2 + \ldots + x_n$, we should have $\delta(x) = \Delta(x_1) + \Delta(x_2) + \ldots + \Delta(x_n)$.
- Substituting $x = x_1 + x_2 + \ldots + x_n$ into this formula, we get

$$\delta(x_1 + x_2 + \ldots + x_n) = \Delta(x_1) + \Delta(x_2) + \ldots + \Delta(x_n).$$

- We do not know a priori what will be the values x_i.
- We want to maintain the desired accuracy $\delta(x)$ – and make sure that we do not get more accuracy.
- So, we should make sure that the equality be satisfied for all possible values $x_1, x_2, \ldots, x_n$.

19. Let us describe what we want in precise terms (cont-d)

- In these terms, the problem takes the following form:
  - for which functions $\delta(x)$ is it possible to have a function $\Delta(x)$ for which the above equality is satisfied? and
  - how can we find this function $\Delta(x)$ – that describes the corresponding measuring instrument?
- This is the problem that we solve in this talk.

20. When Is Optimal Distributive Measurement of Cumulative Quantities Possible?

- We assumed that the function $\Delta(x)$ is smooth, i.e., differentiable.
- Thus, the sum $\delta(x)$ of such functions is differentiable too.
- Since both functions $\Delta(x)$ and $\delta(x)$ are differentiable, we can differentiate both sides of the above equality with respect to x_1.
- The terms $\Delta(x_2), \ldots, \Delta(x_n)$ do not depend on x_1 at all, so their derivative with respect to x_1 is 0.
- Thus, the resulting formula takes the form

$$\delta'(x_1 + x_2 + \ldots + x_n) = \Delta'(x_1).$$

- Here, as usual, δ' and Δ' denote the derivatives of the corresponding functions.
- The new equality holds for all possible values $x_2, \ldots, x_n$.
- For every real number x_0, we can take, e.g., $x_2 = x_0 - x_1$ and $x_3 = \ldots = x_n = 0$, then we will have $x_1 + x_2 + \ldots + x_n = x_0$.

21. When Is Optimal Distributive Measurement of Cumulative Quantities Possible (cont-d)

- So, the equality takes the form $\delta'(x_0) = \Delta'(x_1)$.
- The right-hand side does not depend on x_0, which means that the derivative $\delta'(x_0)$ is a constant not depending on x_0 either.
- The only functions whose derivative is a constant are linear functions.
- So we conclude that the dependence $\delta(x)$ is linear: $\delta(x) = a + b \cdot x$ for some constants a and b.
- Interestingly, this fits well with the usual description of measurement error, as consisting of two components:
  - the absolute error component a that does not depend on x at all,
  - and the relative error component.

22. When Is Optimal Distributive Measurement of Cumulative Quantities Possible (cont-d)

- For relative error, the bound on the measurement error is a certain percentage of the actual value x.
- So, it has the form $b \cdot x$ for some constant b.
- Example: 10% accuracy means $b = 0.1$. Thus, our result explains this usual description.

23. What Measuring Instrument Should We Select to Get the Optimal Distributive Measurement?

- Now we know for what desired accuracy $\delta(x)$, we can have the optimal distributive measurement of a cumulative quantity.
- The natural next question is:
  - given one of such functions $\delta(x)$,
  - what measuring instrument – i.e., what function $\Delta(x)$ – should we select for this optimal measurement?
- To answer this question, we can take $x_1 = \ldots = x_n$.
- In this case, $\Delta(x_1) = \ldots = \Delta(x_n)$, so the above equality takes the form $\delta(n \cdot x_1) = n \cdot \Delta(x_1)$.
- We know that $\delta(x) = a + b \cdot x$, so $a + b \cdot n \cdot x_1 = n \cdot \Delta(x_1)$.

24. What Measuring Instrument Should We Select to Get the Optimal Distributive Measurement (cont-d)

- If we divide both sides of this equality by n, and rename x_1 into x, we get the desired expression for $\Delta(x)$: $\Delta(x) = \dfrac{a}{n} + b \cdot x$.
- In other words:
  - the bound on the relative error component of each measuring instrument should be the same as for the cumulative quantity;
  - the bound on the absolute error component should be n times smaller than for the cumulative quantity.

25. THIRD CHALLENGE: a brief description

- Measurements are never absolutely accurate.
- The measurement result $\widetilde{x}$ is, in general, different from the actual (unknown) value x: $\Delta x = \widetilde{x} - x \neq 0$.
- If there is a bias, i.e., if the mean value $E[\Delta x]$ is non-zero, we can subtract this bias and get $E[\Delta x] = 0$.
- So, we can safely assume that there is no bias.
- For each measuring instrument, we know the standard deviation σ_i of the measurement error.
- Usually, we apply an algorithm $f(x_1, \ldots, x_n)$ to process measurement results.
- Ideally, we should get $y = f(x_1, \ldots, x_n)$, but we only get

$$\widetilde{y} = f(\widetilde{x}_1, \ldots, \widetilde{x}_n).$$

26. Third challenge: a brief description

- How big is the approximation error $\Delta y = \widetilde{y} - y$?
- Measurement errors Δx_i are usually small.
- So, we can:
  - expand the dependence in Taylor series, and
  - ignore terms which are quadratic or higher order in terms of Δx_i.
- As a result, we get a linear approximation $\Delta y = \sum\limits_{i=1}^{n} c_i \cdot \Delta x_i$, where $c_i \stackrel{\text{def}}{=} \dfrac{\partial f}{\partial x_i}$.
- Here, the mean value of Δy is 0.
- The standard deviation σ depends on whether the measurement errors are independent.
- If all Δx_i are independent, then $\sigma^2 = \sum\limits_{i=1}^{n} c_i^2 \cdot \sigma_i^2$.

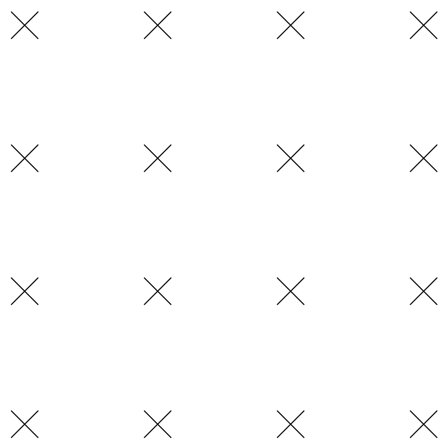
27. Third challenge: a brief description

- If we have no information about independence, then the largest possible value σ is $\sum\limits_{i=1}^{n} |c_i| \cdot \sigma_i$.
- But what if most measurement errors are independent, but for a few, we do not know?
- Or, vice versa, we only known independence of a few pairs?
- Such exceptional pairs naturally form a graph.
- In Chapter 4, we provide expressions for σ for cases when this graph has ≤ 4 elements.
- For example, if all pairs are independent except for $(\Delta x_1, \Delta x_2)$, then

$$\sigma^2 = \sum_{i=3}^{n} c_i^2 \cdot \sigma_i^2 + (|c_1| \cdot \sigma_1 + |c_2| \cdot \sigma_2)^2.$$

28. FOURTH CHALLENGE: description of the case study

- The main idea is to set up a lattice of sensors that would monitor the electric grid.
- Based on the measurement results provided by the sensors:
  - we would get a good picture of the current state of the grid, and
  - we would be able to effectively control it.



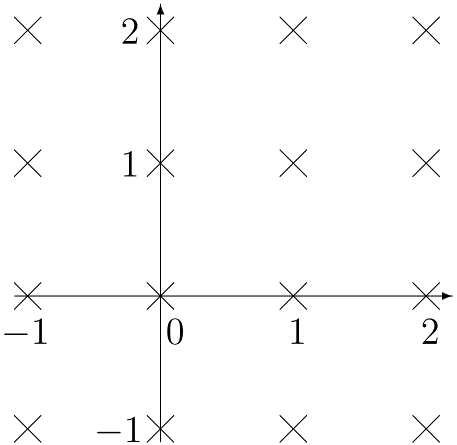
29. How the grid of sensor can detect faults

- Each sensor measures characteristics of the electric current at its location.
- Each fault affects all the sensors, some more, some less.
- By observing the changes in the sensor signals, we can detect the existence of the fault.
- We can also get some information of the fault's location.
- Sensors closer to the fault's location will detect a stronger change in their measurements results than sensors which are further away.
- Thus, by comparing the measurement results of the two sensors, we can decide whether the fault is:
  - closer to the first sensor or
  - closer to the second sensor.

30. Let us describe this situation in precise terms

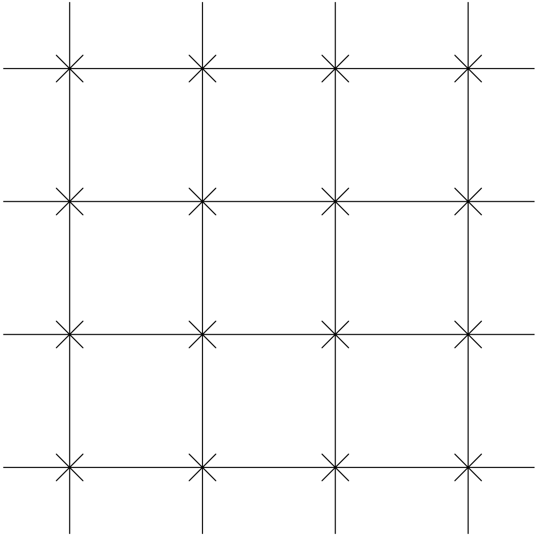
- Let us consider the case when the sensors form a (potentially infinite) rectangular lattice.
- For simplicity of analysis, let us select a coordinate system in which:
  - the location of one the sensors is the starting point $(0,0)$, and
  - the distance between the closest sensors is used as a measuring unit.
- In this coordinate system, sensors are located at all the points (a,b) with integer coordinates.

31. Let us describe this situation in precise terms (cont-d)



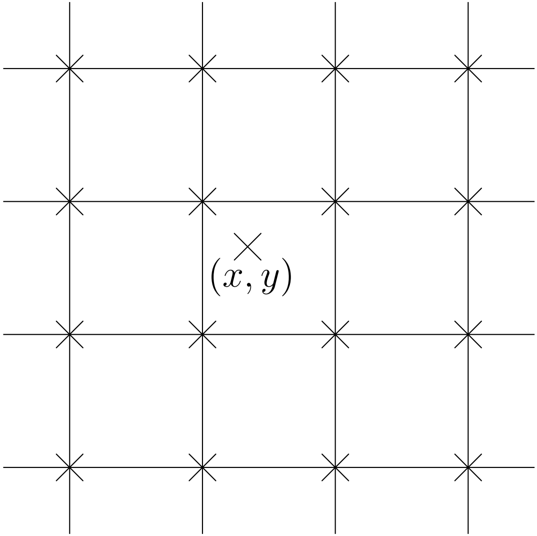
32. Let us describe this situation in precise terms (cont-d)

- These sensors divide the plane into squares $[a, a+1] \times [b, b+1]$.



33. Let us describe this situation in precise terms (cont-d)

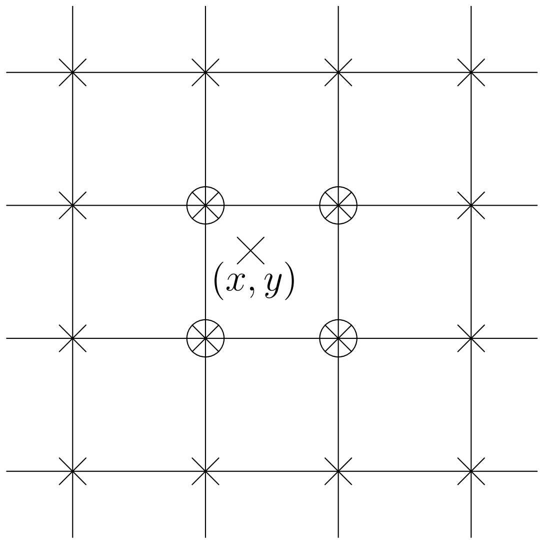
- Each spatial location (x, y) is in one of these squares.



34. Let us describe this situation in precise terms (cont-d)

- One can easily check that:
  - for each spatial location within a square,
  - the vertices (a, b), $(a, b+1)$, $(a+1, b)$, and $(a+1, b+1)$ of this square are the closest grid points.
- Thus:
  - by finding the 4 sensors at which the disturbance signal is the strongest,
  - we can find the square that contains the location of the fault.

35. Let us describe this situation in precise terms (cont-d)



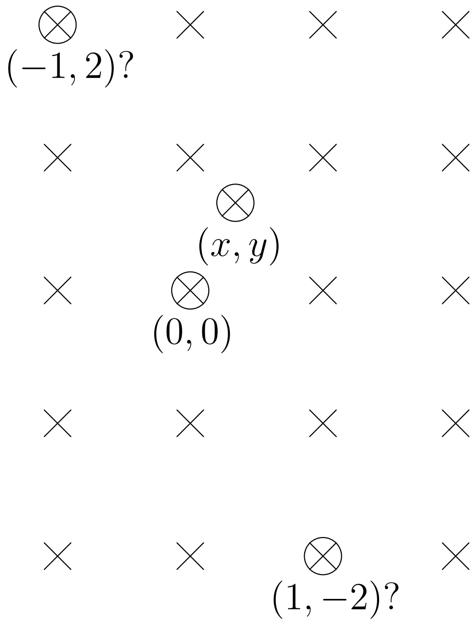
36. Research question

- Can we determine the location of the fault more accurately than "somewhere in the square"?

37. Our answer

- We show that, in principle:
  - by using the lattice of sensors,
  - we can locate the fault with any desired accuracy.
- Indeed, without losing generality, let us assume that the square containing the fault is the square $[0, 1] \times [0, 1]$.
- In other words, we know that the coordinates (x, y) of the fault satisfy the inequalities $0 \le x \le 1$ and $0 \le y \le 1$.
- For each pair of positive integers (p, q), we can check whether
  - the sensor at $(p, -q)$ gets a stronger signal than
  - the sensor at $(-p, q)$.

38. Our answer (cont-d)

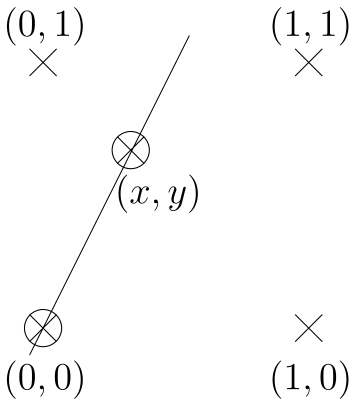


39. Our answer (cont-d)

- The first sensor's signal is stronger if and only if:
  - the squared distance $d^2(f,s_1)=(x-p)^2+(y-(-q))^2$ between the fault f and the first sensor s_1 is smaller than
  - the squared distance $d^2(f,s_2)=(x-(-p))^2+(y-q)^2$ to the second sensor.
- One can check that $d^2(f,s_1)<d^2(f,s_2)$ if and only if $q\cdot y<p\cdot x$, i.e., if and only if $y/x<p/q$.
- A real number can be uniquely determined if we know:
  - which rational numbers p/q are smaller than this number and
  - which are larger.
- Thus:
  - by comparing signals from different sensors,
  - we can determine the ratio $r\stackrel{\rm def}{=}y/x$ with any given accuracy.

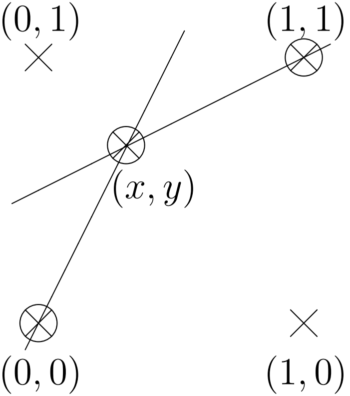
40. Our answer (cont-d)

- Hence, we can determine the line $y = r \cdot x$ going through $(0, 0)$ that contains the fault.



41. Our answer (cont-d)

- Similarly, we can find a straight line going through the point $(1, 1)$ that contains the fault.
- Thus:
  - the fault's location can be uniquely determined
  - as the intersection of these two straight lines.



42. Thanks

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