Towards Combining Probabilistic, Interval, Fuzzy Uncertainty, and Constraints: On the Example of Inverse Problem in Geophysics

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Determining Earth . . .
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1. Determining Earth Structure Is Very Important

- *Importance:* civilization greatly depends on the things we extract from the Earth: oil, gas, water.
- *Need* is growing, so we must find new resources.
- *Problem:* most easy-to-access mineral resources have been discovered.
- *Example:* new oil fields are at large depths, under water, in remote areas – so drilling is very expensive.
- *Objective:* predict resources before we invest in drilling.
- *How:* we know what structures are promising.
- *Example:* oil and gas concentrate near the top of (natural) underground domal structures.
- *Conclusion:* to find mineral resources, we must determine the structure at different depths z at different locations (x, y).

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2. Data that We Can Use to Determine the Earth Structure

- *Available measurement results:* those obtained without drilling boreholes.
- *Examples:*
  - gravity and magnetic measurements;
  - travel-times t_i of seismic ways through the earth.
- *Need for active seismic data:*
  - passive data from earthquakes are rare;
  - to get more information, we make explosions, and measure how the resulting seismic waves propagate.
- *Resulting seismic inverse problem:*
  - we know the travel times t_i;
  - we want to reconstruct velocities at different depths.

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3. Algorithm for the Forward Seismic Problem

- *We know:* velocities v_j in each grid cell j.
- *We want to compute:* traveltimes t_i.
- *First step:* find shortest (in time) paths.
- *Within cell:* path is a straight line.
- *On the border:* between cells with velocities v and v', we have Snell's law $\dfrac{\sin(\varphi)}{v} = \dfrac{\sin(\varphi')}{v'}$.
- *Comment:* if $\sin(\varphi') > 1$, the wave cannot get penetrate into the neighboring cell; it bounces back.
- *Resulting traveltimes:* $t_i = \sum\limits_j \dfrac{\ell_{ij}}{v_j}$, where ℓ_{ij} is the length of the part of i-th path within cell j.
- *Simplification:* use slownesses $s_j \stackrel{\text{def}}{=} \dfrac{1}{v_j}$; $t_i = \sum\limits_j \ell_{ij} \cdot s_j$.

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4. Algorithm for the Inverse Problem: General Description

- *The most widely used:* John Hole's iterative algorithm.
- *Starting point:* reasonable initial slownesses.
- *On each iteration:* we use current (approximate) slownesses s_j to compute the travel-times $t_i = \sum_j \ell_{ij} \cdot s_j$.
- *Fact:* measured travel-times $\widetilde{t}_i$ are somewhat different: $\Delta t_i \stackrel{\text{def}}{=} \widetilde{t}_i - t_i \neq 0$.
- *Objective:* find Δs_j so that $\sum \ell_{ij} \cdot (s_j + \Delta s_j) = \widetilde{t}_i$.
- *Problem:* we have many observations n, and computation time $\sim n^3$ – too long, so we need faster techniques.
- *Stopping criterion:* when average error $\dfrac{1}{n} \sum_{i=1}^{n} (\Delta t_i)^2$ is below noise.

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5. Algorithm for the Inverse Problem: Details

- *Objective (reminder):* find Δs_j s.t. $\sum \ell_{ij} \cdot \Delta s_j = \Delta t_i$.
- *Simplest case:* one path.
- *Specifics:* under-determined system: 1 equation, many unknowns Δs_j.
- *Idea:* no reason for Δs_j to be different: $\Delta s_j \approx \Delta s_{j'}$.
- *Formalization:* minimize $\sum\limits_{j,j'} (\Delta s_j - \Delta s_{j'})^2$ under the constraint $\sum \ell_{ij} \cdot \Delta s_j = \Delta t_i$.
- *Solution:* $\Delta s_j = \dfrac{\Delta t_i}{L_i}$ for all j, where $L_i = \sum\limits_j \ell_{ij}$.
- *Realistic case:* several paths; we have Δs_{ij} for different paths i.
- *Idea:* least squares $\sum\limits_i (\Delta s_j - \Delta s_{ij})^2 \to \min$.
- *Solution:* Δs_j is the average of Δs_{ij}.

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6. Successes, Limitations, Need for Prior Knowledge

- *Successes:* the algorithm usually leads to reasonable geophysical models.
- *Limitations:* often, the resulting velocity model is not geophysically meaningful.
- *Example:* resulting velocities outside of the range of reasonable velocities at this depth.
- *It is desirable:* incorporate the expert knowledge into the algorithm for solving the inverse problem.

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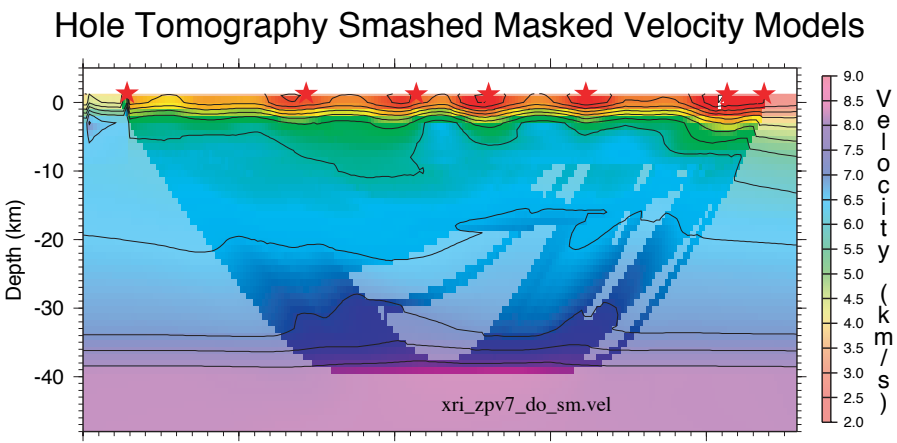

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7. Case of Interval Prior Knowledge

- *Idea:* for each cell j, a geophysicist provides an interval $[\underline{s}_j, \overline{s}_j]$ of possible values of s_j.
- *Hole's code:* along each path i, we find corrections Δs_{ij} that minimize

$$\sum_{j,j'} (\Delta s_{ij} - \Delta s_{ij'})^2$$

under the constraint

$$\sum_{j=1}^{c} \ell_{ij} \cdot \Delta s_{ij} = \Delta t_i.$$

- *Modification:* we must minimize under the additional constraints

$$\underline{s}_j \le s_j^{(k)} + \Delta s_{ij} \le \overline{s}_j.$$

- *What we designed:* an $O(c \cdot \log(c))$ algorithm for solving this new problem.

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8. New Algorithm: For Each Path on Each Iteration

- *Case:* $\Delta t_i > 0$; for $\Delta t_i < 0$, we have similar formulas.
- Compute, for each cell j,

$$\underline{\Delta}_j = \underline{s}_j - s_j^{(k-1)} \text{ and } \overline{\Delta}_j = \overline{s}_j - s_j^{(k-1)}.$$

- Sort values $\overline{\Delta}_j$ into

$$\overline{\Delta}_{(1)} \le \overline{\Delta}_{(2)} \le \ldots \le \overline{\Delta}_{(c)}.$$

- For every p from 0 to c, compute:

$$A_0 = 0, \ \ \mathcal{L}_0 = L_i, \ \ A_p = A_{p-1} + \ell_{i(p)} \cdot \overline{\Delta}_{(p)}, \ \ \mathcal{L}_p = \mathcal{L}_{p-1} - \ell_{i(p)}.$$

- Compute $S_p = A_p + \mathcal{L}_p \cdot \Delta_{(p+1)}$, and find p s.t.

$$S_{p-1} \le \Delta t_i < S_p.$$

- Take $\Delta s_{i(j)} = \overline{\Delta}_j$ for $j \le p$, and $\Delta s_{(j)} = \dfrac{\Delta t_i - A_p}{\mathcal{L}_p}$ else.
- Then, average Δs_{ij} over paths i.

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9. Explicit Expert Knowledge: Fuzzy Uncertainty

- Experts can usually produce an wider interval of which they are practically 100% certain.
- In addition, experts can also produce narrower intervals about which their degree of certainty is smaller.
- As a result, instead of a *single* interval, we have a *nested* family of intervals corresponding to different levels of uncertainty.
- In effect, we get a *fuzzy* interval (of which different intervals are α-cuts).
- *Previously:* a solution is satisfying or not.
- *New idea:* a satisfaction *degree* d.
- *Specifics:* d is the largest α for which all s_i are within the corresponding α-cut intervals.

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10. How We Can Use Fuzzy Uncertainty

- *Objective:* find the largest possible value α for which the slownesses belong to the α-cut intervals.
- *Possible approach:*
  - try $\alpha = 0$, $\alpha = 0.1$, $\alpha = 0.2$, etc., until the process stops converging;
  - the solution corresponding to the previous value α is the answer.
- *Comment:*
  - this is the basic straightforward way to take fuzzy-valued expert knowledge into consideration;
  - several researchers successfully used fuzzy expert knowledge in geophysics (Nikravesh, Klir, et al.);
  - we plan to add their ideas to our algorithms.

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11. Case of Probabilistic Prior Knowledge

- *Description:* from prior observations, we know $\widetilde{s}_j \approx s_j$, and we know the st. dev. σ_j of this value.
- *Minimize:* $\sum_{j,j'} (\Delta s_{ij} - \Delta s_{ij'})^2$ s.t. $\sum_{j=1}^{c} \ell_{ij} \cdot \Delta s_{ij} = \Delta t_i$ and

$$\frac{1}{n} \cdot \sum_{j=1}^{c} \frac{((s_j^{(k)} + \Delta s_{ij}) - \widetilde{s}_j)^2}{\sigma_j^2} = 1.$$

- *Solution* (Lagrange multipliers): $\overline{\Delta s} \stackrel{\text{def}}{=} \frac{1}{n} \cdot \sum_{j=1}^{c} \Delta s_{ij}$,

$$\frac{2}{n} \cdot \Delta s_{ij} - \frac{2}{n} \cdot \overline{\Delta s} + \lambda \cdot \ell_{ij} + \frac{2\mu}{n \cdot \sigma_j^2} \cdot (s_j^{(k)} + \Delta s_{ij} - \widetilde{s}_j) = 0.$$

- *Fact:* Δs_{ij} is an explicit function of λ, μ, $\overline{\Delta s}$.
- *Algorithm:* solve 3 non-linear equations (above one + 2 constraints) with unknowns λ, μ, $\overline{\Delta s}$.

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12. Combination of Different Types of Prior Knowledge

- *Need:* we often have both:
  - prior measurement results – i.e., *probabilistic* knowledge, and
  - expert estimates – i.e., *interval* and *fuzzy* knowledge.
- *Minimize:* $\sum_{j,j'}(\Delta s_{ij} - \Delta s_{ij'})^2$ s.t. $\sum_{j=1}^{c} \ell_{ij} \cdot \Delta s_{ij} = \Delta t_i$,

$$\frac{1}{n} \cdot \sum_{j=1}^{c} \frac{((s_j^{(k)} + \Delta s_{ij}) - \widetilde{s}_j)^2}{\sigma_j^2} \le 1,$$

  and $\underline{s}_j \le s_j^{(k)} + \Delta s_{ij} \le \overline{s}_j$.
- *Idea:* we minimize a convex function under convex constraints; efficient algorithms are known.

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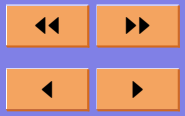
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13. Combination of Different Types of Prior Knowledge: Algorithm

- *Idea* – method of alternating projections:
  - first, add a correction that satisfy the first constraint,
  - then, the additional correction that satisfies the second constraint,
  - etc.
- *Specifics:*
  - first, add equal values Δs_{ij} to minimize Δt_i;
  - restrict the values to the nearest points from $[\underline{s}_j, \overline{s}_j]$,
  - find the extra corrections that satisfy the probabilistic constraint,
  - repeat until converges.

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