Interval computations in the metrology

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Introduction: measurements

$\rightarrow$ **The main property of any measurement result is its uncertainty or error**. It is the main quality parameter for performed measurement.

$\rightarrow$ Let us measure some voltage quantity x_{real}. Let us receive $x_{measured}$ = 1.05 V from measuring system. **Is it close to the real value** x_{real}? To answer how accurate it is, one has to estimate its absolute error $\Delta x = x_{measured} - x_{real}$.

Introduction: measurements
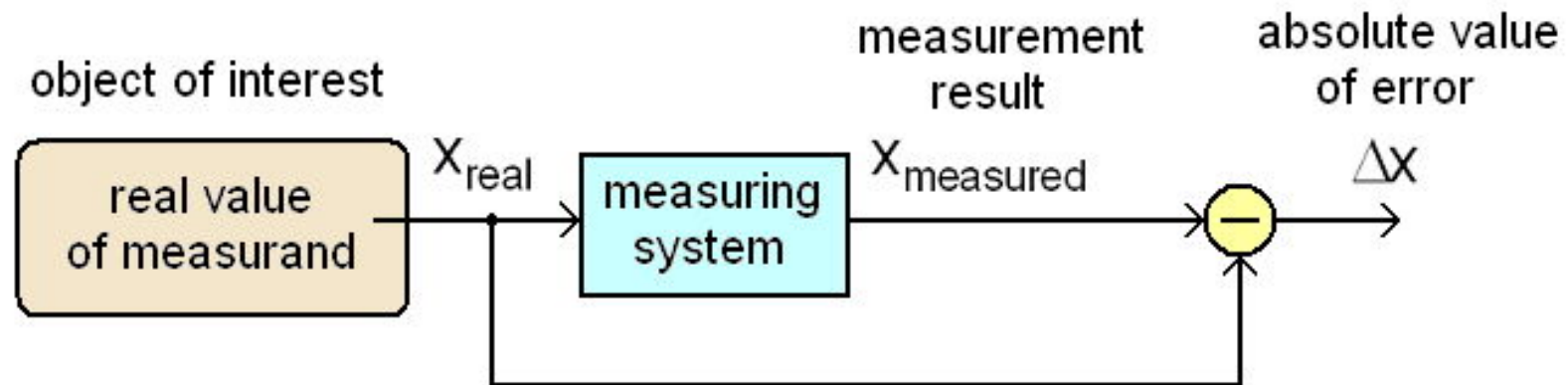
$\rightarrow$ **We never know the measurand real value** x_{real}.
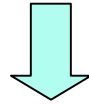
We never know the error value Δx. The only thing we can do is to use interval $\left[\underline{\Delta x}, \overline{\Delta x}\right]$ of its possible values. Its bounds can be retrieved from technical documentation for used measuring instrument.
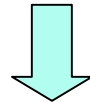
If we process data obtained from measurements then we will get in principle inaccurate results. We need always to estimate the transformed uncertainty inherited from the initial data errors. Otherwise we will incorrect in our further reasoning that will be based on these results.
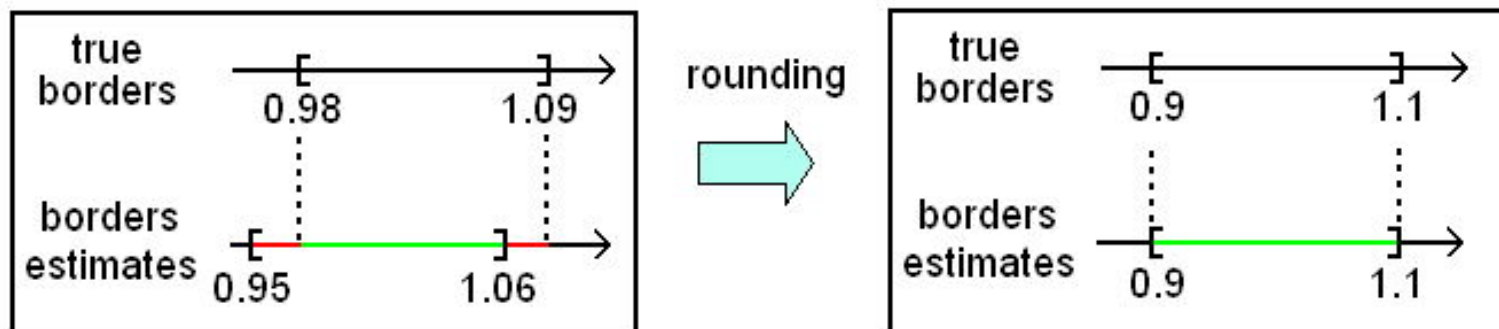
Measurements results processing

$\rightarrow$ **Metrological case is specific.**

Classical approaches for uncertainty propagation always provide bounds $J[\Delta x]$ for estimated interval $\left[\underline{\Delta x}, \overline{\Delta x}\right]$ that guarantees its coverage: $J \supseteq \left[\underline{\Delta x}, \overline{\Delta x}\right]$.
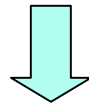
As a conclusion J is almost always overestimated, sometimes catastrophically.

$\rightarrow$ **In metrology we can allow J to be slightly over- or even slightly underestimated because of results' rounding.**

3

Measurements results processing

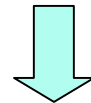$\rightarrow$ **Linearization can be used.**

Let $y = f(x_1, ..., x_n)$ be a function to process the measurement results $x_1, ..., x_n$. Then

$$\Delta y \approx \sum_{i=1}^{n} \frac{\partial f(x_1, ..., x_n)}{\partial x_i} \cdot \Delta x_i$$

Function f is determined by computer program. To obtain its partial derivatives we can use **automatic differentiation** technique.

! We can take into consideration only linear operations with measurement errors in interval computations in metrology.

4

The heat consumption metering

$\rightarrow$ **One of the most natural examples for interval computations applying in metrology is the heat consumption metering.**

The Part 1 of the acting European standard EN-1434 introduces the following simple formula to determine the heat consumption during the time interval from τ_0 to τ_1:

$$Q = \int_{\tau_0}^{\tau_1} M(\tau) \cdot \Delta h(\tau) d\tau$$

where Q is the consumed heat quantity, $M(\tau)$ is the mass flow-rate of the heat-carrier (water) in the time moment τ, $\Delta h(\tau)$ is the enthalpies difference between hot water on the house heating system inlet and cooled water on its outlet.

5

The heat consumption metering

$\rightarrow$ **All the variables included in the formula for the heat quantity are the measurements results and are inaccurate.**
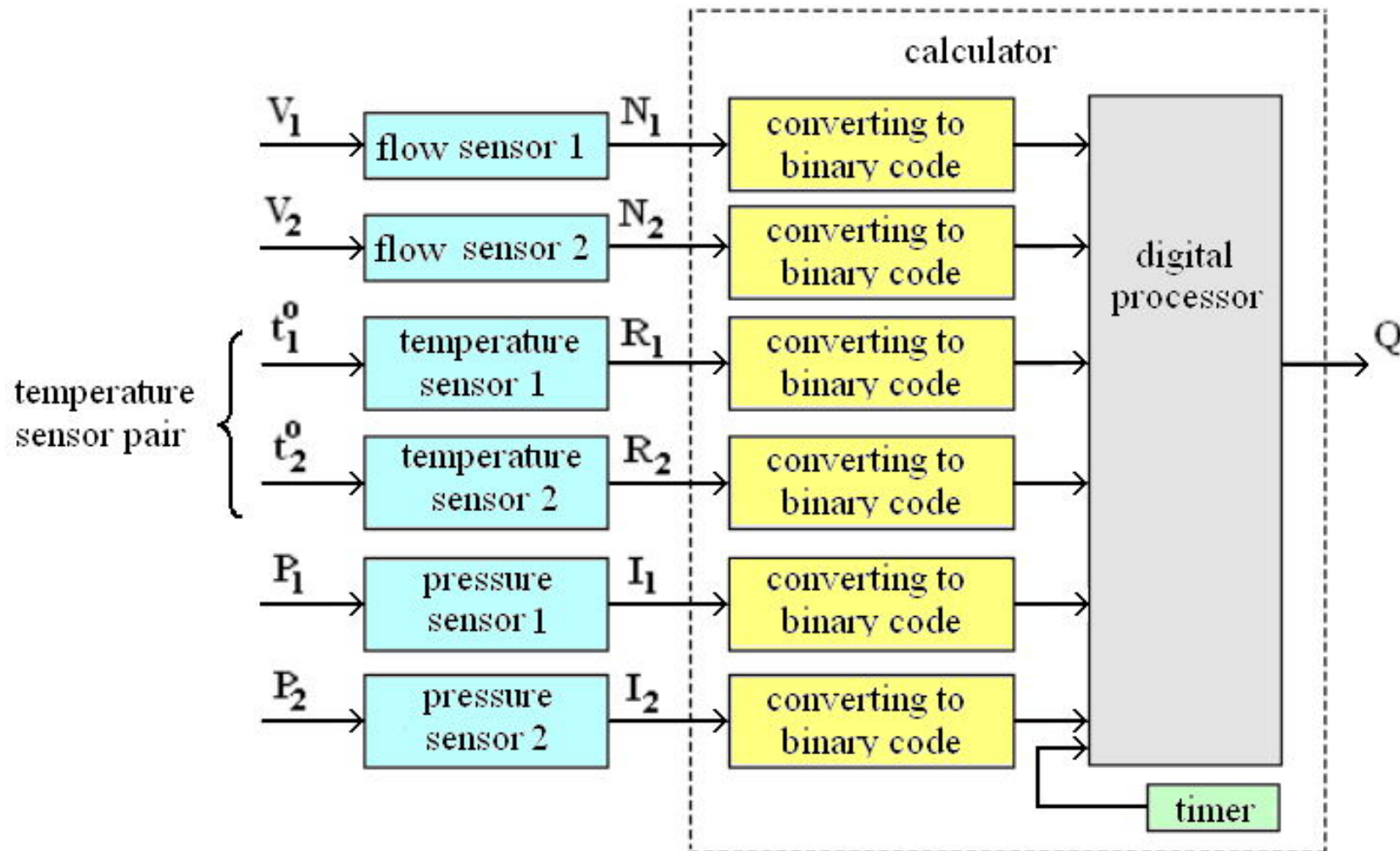
The typical structure of heat-metering system includes 6 sensors. The typical heat exchange circuit is below:
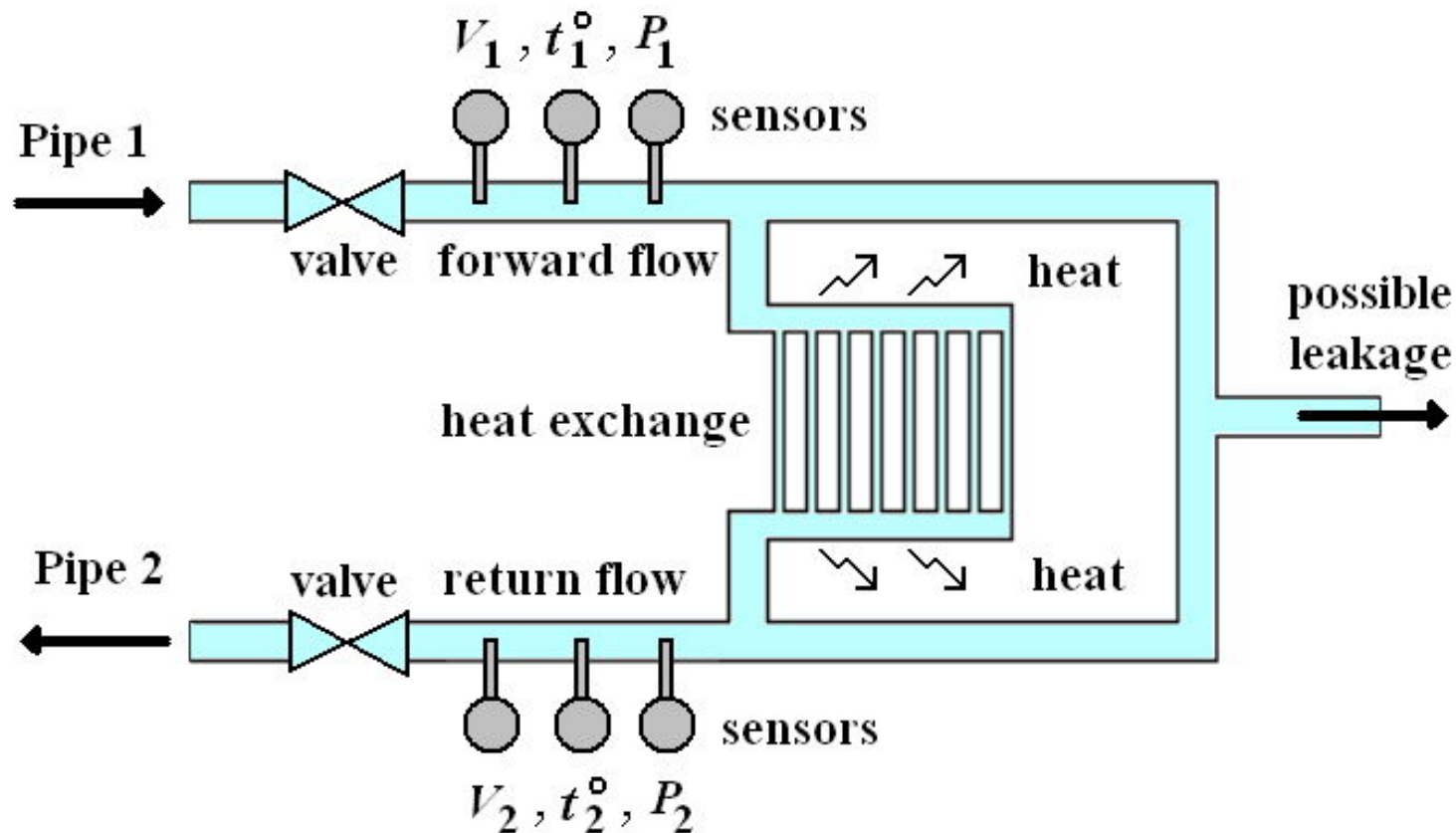
The heat consumption metering

$\rightarrow$ **Typical heat-meter structural scheme contains digital data processor that calculates the heat quantity.**

7

→ **Each of used sensors passed the metrological tests and has the certificate on its accuracy. The obtained values of the direct measurements are used to calculate the consumed heat quantity.**

The corresponding computing of the heat quantity Q is processed on the base of the legislatively fixed formulas, like this:

$$Q = M_1 \cdot (h_1 - h_2) + (M_1 - M_2) \cdot (h_2 - h_c)$$

where $M_1 = \rho_1 \cdot V_1$, $M_2 = \rho_2 \cdot V_2$ are the accumulated masses of the heat-carrier that passes in the forward and return flows; and ρ_1 and ρ_2 are the densities of the water in these flows; h_1, h_2, h_c are the enthalpy values in the forward and return flows and also the enthalpy of the cold water.

The heat consumption metering

→   **The acting standard EN-1434 contains the following requirements for the accuracy of used sub-assembles.**

«9.2.2 Maximum permissible relative error of sub-assemblies
9.2.2.1 Calculator
$E_c = \pm (0.5 + \Delta\Theta_{min} / \Delta\Theta)$ **!** where other processed measured quantities?
…
9.2.2.2 Temperature sensor pair
$E_t = \pm (0.5 + 3 \Delta\Theta_{min} / \Delta\Theta)$

…
9.2.2.3 Flow sensor
Class 1: $E_f = \pm (1 + 0.01 \, q_p/q)$, but not more than $\pm 3.5 \%$.
Class 2: $E_f = \pm (2 + 0.02 \, q_p/q)$, but not more than $\pm 5 \%$.
Class 3: $E_f = \pm (3 + 0.05 \, q_p/q)$, but not more than $\pm 5 \%$».

In the cited fragment, $\Delta\Theta_{min} = 3\ °C$, q is the measured value of the water flow and q_p is the maximum permissible value of the water flow range. Unhopefully, the list of the sub-assemblies doesn't refer to the analog-to-digital converters

9

The heat consumption metering

$\rightarrow$ **The essential defect of the standard EN-1434 is that it demands to compute the arithmetic sum of the relative errors of the initial data used to calculate the consumed heat quantity *Q*. This is incorrect.**

Here is the referred fragments of the standard.

«**9.3.2** For a combination of sub-assemblies as defined in 3.5, the maximum permissible error for the combination is the arithmetic sum of the maximum permissible errors of each sub-assembly.

9.3.3 The errors of combined instruments shall not exceed the arithmetic sum of the maximum permissible errors of the sub-assemblies indicated in 9.2.2.1 to 9.2.2.3».

The typically used formula for the heat quantity is

$$Q = M_1 \cdot (h_1 - h_2) + (M_1 - M_2) \cdot (h_2 - h_c)$$
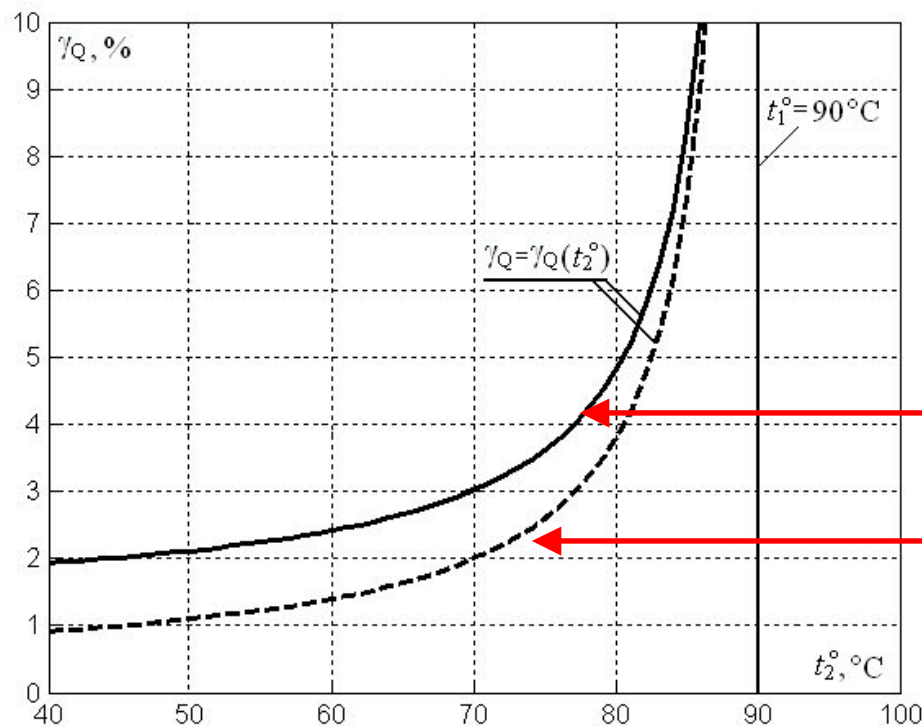
$$\Downarrow$$
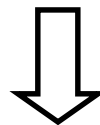
$$\Delta Q \neq \Delta M_1 + \Delta M_2 + \Delta h_1 + \Delta h_2 + \Delta h_c \quad !$$

$$\gamma_Q \neq \gamma_{M_1} + \gamma_{M_2} + \gamma_{h_1} + \gamma_{h_2} + \gamma_{h_c} \quad !$$

10

The heat consumption metering

$\rightarrow$ **If we take into account the inaccuracy only of the part of the initial data then we underestimate the uncertainty of the consumed heat quantity**

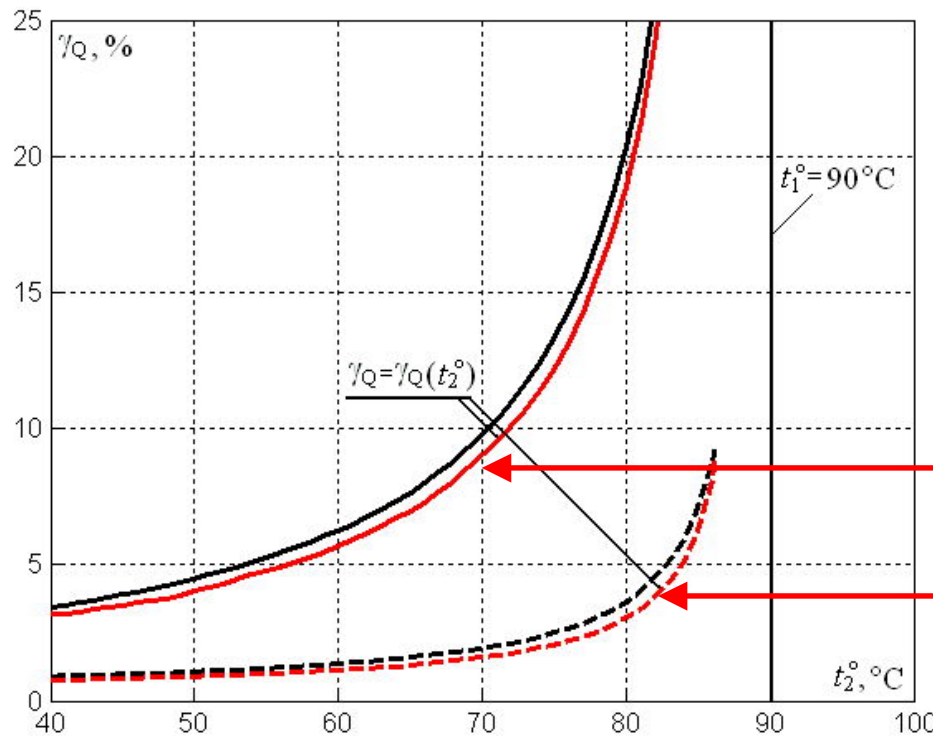Below the results of the numeric examples are presented obtained with interval computations.

If we take into account all data uncertainty

If we take into account only temperature uncertainty as it is recommended in EN-1434

11

The heat consumption metering

→ **One more numeric example (for the case when there are leakages in hot water heating)**

We see drastic underestimation of the consumed heat quantity uncertainty.

Red lines are the results of Monte-Carlo tests to check the reliability of obtained results

If we take into account all data uncertainty

If we take into account only temperature uncertainty as it is recommended in EN-1434

12

Thank you for attention!