Creative Discussions or Memorization? Maybe Both? (on the example of teaching Computer Science)

Olga Kosheleva[1] and Vladik Kreinovich[2]
[1]Department of Teacher Education
[2]Department of Computer Science
University of Texas at El Paso, El Paso, TX 79968, USA
olgak@utep.edu, vladik@utep.edu

1. Outline

- We all strive to be creative in our teaching.
- However, there is often not enough time to make all the topics creative fun.
- So sometimes, we teach memorization first, understanding later.
- We do it, but we often do it without seriously analyzing which topics to "sacrifice" to memorization.
- In this talk, we use simple mathematical models of learning to come up with relevant recommendations.
- Namely, all the topics form a dependency graph.
- The most reasonable topics for memorization first are the ones in the critical path of this graph.

2. Creativity Good, Memorization Bad

- Modern pedagogical literature is very convincing:
  - creative discussions lead to a better understanding
  - than memorization.
- Gently guided by an instructor, students
  - solve interesting problems and
  - uncover – themselves – the desired formula.
- This is great:
  - the students fell good about it,
  - they remember it better,
  - they use it more creatively.

3. And Yet, and Yet ...

- Some students of introductory CS cannot move forward since they forgot a formula for the log of the product.
- Some forgot even how to add fractions.
- Yes, we can stop and let them recreate this formula – but:
  - do we really want to teach a few weeks less computing and a few weeks more math?
  - and are we, CS folks, the best teachers of math?

4. What We Do

- What many of us do is:
  - have students memorize the needed math and
  - use the remaining time to be creative in computing.
- Even in computing:
  - we ask students to memorize patterns corresponding to sum, maximum, etc.,
  - instead of having them re-create all these codes creatively every time.
- We do it, but we do it shamefully: should not everything in education be creative fun?
- Our point is: maybe we should not feel guilty.
- In this talk, we justify our point by analyzing simple mathematical models of teaching.

5. Informal (Qualitative) Analysis of the Problem

- Our first argument is that:
  - while creative teaching is good,
  - it is often slower.
- In most classes, there is a dependence between material:
  - to study some topics,
  - students need to know some previous ones.
- In the resulting dependence, there is often a critical path.
- Along this path, it may be better to use memorization first – and get a deep understanding later.
- Another argument is that we want to optimally use the student's brains.

6. Analysis of the Problem (cont-d)

- Yes, it would be nice if we could keep the brains in the permanent state of active creative fun.
- However, brains get tired, they need rest.
- Here, memorization helps.
- To solve a non-trivial problem, we use creative thinking to find known patterns for solve it.
- Then we "switch off" the active brain and use memorized techniques to solve the resulting subproblems.
- If we end up with a quadratic equations, we do not want to recall the tricks that lead to the formulas.
- We just want to plug in the numbers.
- Meanwhile, the active brain rests and gets ready for new creative activities – and everyone benefits!

7. Formal (Quantitative) Analysis of the Problem

- Let us denote the total amount of creative effort that a student can perform during the learning period by E.
- We want to have the best overall learning result.
- What is the proper way to distribute this amount between different moments of time?
- Let n denote the overall number of moment of time.
- Let e_i denote the amount of creative effort that a student uses at moment i.
- Let $r(e)$ denote the amount of learning that results when a student uses a creative effort e.
- In these terms, we want to maximize
  - the overall results, i.e., the sum $r(e_1) + \ldots + r(e_n)$,
  - under the constraint that the overall creative effort $e_1 + \ldots + e_n$ is equal to the given amount E.

8. Solving the Problem

- We want to find the values $e_1, \ldots, e_n$ that

$$\text{Maximize } r(e_1) + \ldots + r(e_n)$$

$$\text{under the constraint } e_1 + \ldots + e_n = E.$$

- Lagrange multiplier technique leads to

$$r(e_1) + \ldots + r(e_n) + \lambda \cdot (e_1 + \ldots + e_n - E) \to \max .$$

- Differentiating relative to e_i and equating the derivative to 0, we get $F(e_i) = 0$, where we denoted

$$F(e) \stackrel{\text{def}}{=} r'(e) + \lambda.$$

- Intuitively:
  - small changes in the amount of creative effort e
  - shouldn't drastically affect the learning result $r(e)$.

9. $F(e)$ Should Be Analytical

- Therefore, it is reasonable to assume that the function $r(e)$ is smooth.
- $r(e)$ is probably even analytical (i.e., can be expanded in Taylor series).
- In this case, the function $F(e)$ is also an analytical function.
- It is known that an analytical function $F(e) \not\equiv 0$ can only have finitely many roots on an interval.
- Thus, all the optimal effort amounts e_i must belong to the finite set of these solutions.
- For usual analytical functions, this set of solutions is small.
- Indeed, an arbitrary analytical function, by definition, is equal to its Taylor series.

10. $F(e)$ Should Be Analytical (cont-d)

- An arbitrary analytical function, by definition, is equal to its Taylor series.
- It can therefore be approximated, with an arbitrary accuracy, by a polynomial.
- A polynomial of degree d can have no more than d roots; so, e.g.:
  - if a cubic polynomial is a reasonable approximation for the function $F(e)$,
  - then, in this approximation, the function $F(e)$ has no more than 3 roots.
- So, we use no more than three different levels of creative effort.
- A 7-th order polynomial is usually enough for most known analytical functions such as sin, cos, etc.

11. We Must Alternate Between Higher and Lower Levels of Student Creativity

- This leads to no more than 7 different levels of creative effort; so:
  - in the optimal learning arrangement,
  - we should alternate between a small number of different levels of creativity.
- It is an empirical fact that it is not possible to always maintain the highest level of creativity.
- In our terms, the available amount of effort E is smaller than that.
- So, this means that we do not need to alternate between higher and lower levels of student creativity.

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12. Which Topics Should We Ask Students To Memorize?

- All the topics form a dependency graph.
- We do not have enough time to allow students to treat all topics with equal creativity.
- Thus, the most reasonable topics for memorization first are the ones in the *critical path* of this graph.

13. Other Applications of This Idea to Learning

- A similar argument can be used when:
  - we want to achieve the largest overall result
  - under restrictions on the overall effort.
- The optimal distribution in learning activity is
  - not a steadfast study,
  - but rather periods of intense study separated by periods of relative rest.
- Similarly:
  - the optimal arrangement is not when the teaching efforts are uniformly distributed among students,
  - but rather when there are a few levels and
  - each student is assigned to a certain level (e.g., BSc, MSc, Ph.D.).

14. Applications Beyond Learning

- A biological creature cannot maintain the maximal level of activity.
- Thus, the optimal effect is when a creature alternates between a few levels.
- This explain abrupt transition to sleep, and between sleep phases.
- In control, this explain ubiquity of optimal "bang-bang" control.
- For a person with limited resources, the most satisfactory consumption schedule:
  - is not the schedule in which these resources are equally distributed,
  - but the one with higher ("feasts") and lower ("fasts") consumption periods.

15. Applications Beyond Learning (cont-d)

- In traffic, similar idea explains why the optimal traffic arrangement means that
  - we fix a small number of speed levels, and
  - assign (maybe dynamically) each road to one of these levels.
- In real life, such levels are freeway, city limits, school zone, etc.

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