Reward for Good Performance Works Better Than Punishment for Mistakes: Economic Explanation

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1. Reward vs. Punishment: An Important Economic Problem

- One of the most important issues in economics is how to best stimulate people's productivity.
- What is the best combination of reward and punishment that makes people perform better.
- This problem rises not only in economics, it appears everywhere.
- How do we stimulate students to study better?
- How do we stimulate our own kids to behave better?

2. Empirical Fact

- A lot of empirical studies were done on this topic.
- Some of these studies were made by Nobelist Daniel Kahneman – one of the fathers of behavioral economics.
- Most confirm that reward for good performance, in general, works better than punishment for mistakes.
- But why?
- Like many facts from behavioral economics, this fact does not have a convincing theoretical explanation.
- In this talk, we provide a theoretical explanation for this empirical phenomenon.

3. What People Want

- People spend some efforts e.
- Based on results of these efforts, they get a reward $r(e)$.
- In the first approximation, we can say that the overall gain is the reward minus the efforts: $r(e) - e$.
- A natural economic idea is that every person wants to maximize his/her gain, i.e., maximize $r(e) - e$; so:
  - to explain why rewards work better than punishments,
  - we need to analyze what are the reward functions $r(e)$ corr. to the two reward strategies.
- We will use simplified "first approximation" models, providing qualitative understanding of the situation.

4. What Reward Function Corresponds to Rewarding Good Performance

- What does rewarding good performance mean?
- On the one hand:
  - if the performance is not good, i.e., if the effort e is smaller than the smallest needed effort e_0,
  - there is practically no reward: $r(e) = r_+$ for some

$$r_+ \approx 0.$$

- On the other hand:
  - the more effort the person uses, the larger the reward;
  - so, every effort beyond e_0 is proportionally rewarded, i.e., $r(e) = r_+ + c_+ \cdot (e - e_0)$, for some c_+.

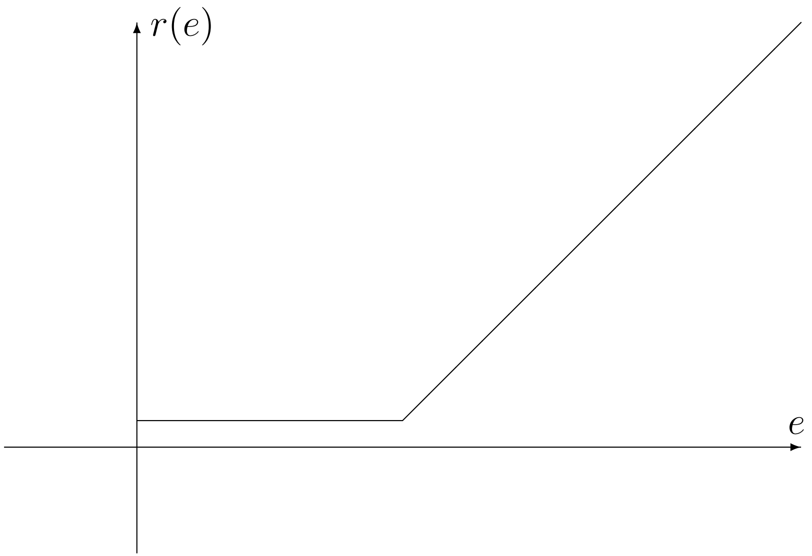
5. Rewarding Good Performance (cont-d)

- The constant c_+ depends on the units used for measuring effort and reward:
  - one unit of effort corresponds
  - to c_+ units of reward.
- These two formulas can be combined into a single formula

$$r(e) = r_+ + \max(0, c_+ \cdot (e - e_0)) = r_+ + c_+ \cdot \max(0, e - e_0).$$

6. Rewarding Good Performance (cont-d)

- This dependence has the following form:



7. What Can We Say About This Function

- It is easy to see that our function is *convex*.
- This means that for all $e' < e''$ and for each $\alpha \in [0, 1]$, we have

$$r(\alpha \cdot e' + (1 - \alpha) \cdot e'') \leq \alpha \cdot r(e') + (1 - \alpha) \cdot r(e'').$$

8. What Reward Function Corresponds to Punishing for Mistakes

- What does punishing for mistakes means?
- On the one hand:
  - if the performance is good, i.e., if the effort e is $\geq$ the smallest needed effort e_0,
  - then there is no punishment, i.e., the reward remains the same: $r(e) = r_-$ for some constant r_-;
- On the other hand:
  - the fewer effort the person uses, the most mistakes he/she makes,
  - so the larger the punishment and the smaller the resulting reward;
  - so, every effort below e_0 is proportionally penalized, i.e., $r(e) = r_- - c_- \cdot (e_0 - e)$, for some c_-.

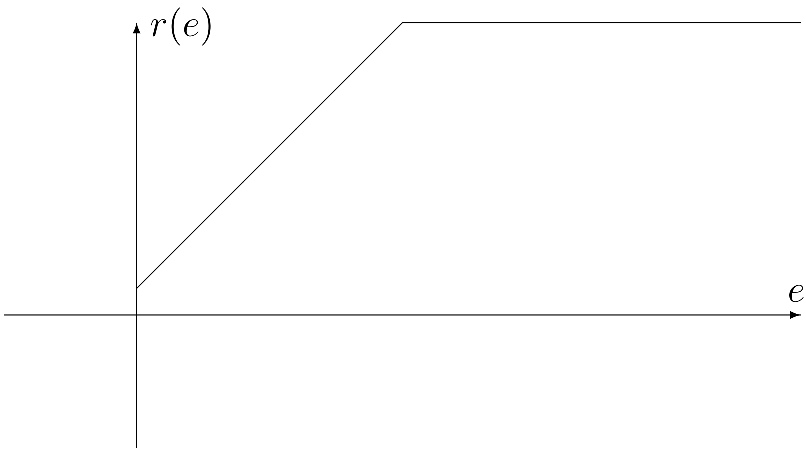
9. Punishing for Mistakes (cont-d)

- The constant c_- depends on the units used for measuring effort and reward:
  - one unit of effort corresponds
  - to c_- units of reward.
- These two formulas can be combined into a single formula

$$r(e) = r_- - c_- \cdot \max(0, e_0 - e) = r_- + c_- \cdot \min(0, e - e_0).$$

10. Punishing for Mistakes (cont-d)

- This dependence has the following form:



11. What Can We Say About This Function

- It is easy to see that this function is *concave*.
- This means that for all $E' < E''$ and for each $\alpha \in [0, 1]$, we have

$$r(\alpha \cdot e' + (1 - \alpha) \cdot e'') \geq \alpha \cdot r(e') + (1 - \alpha) \cdot r(e'').$$

- Now, we are ready to present the desired explanation.

12. Known Properties of Convex and Concave Functions: Reminder

- It is known that:
  - every linear function is both convex and concave;
  - the sum of two convex functions is convex, and
  - the sum of two concave functions is concave.
- In particular, the linear function $f(e) = -e$ is both convex and concave, thus:
  - when the function $r(e)$ is convex, the sum $r(e) + (-e) = r(e) - e$ is also convex; and
  - when the function $r(e)$ is concave, the sum $r(e) + (-e) = r(e) - e$ is also concave.

13. Convex and Concave Functions (cont-d)

- It is also known that:
  - for a convex function, the maximum on an interval is always attained at one of the endpoints;
  - for a concave function, its maximum on an interval is always attained at some point inside the interval.

14. Resulting Explanation

- A person selects the effort e_0 for which the expression $r(e) - e$ attains its largest possible value.
- Of course, people's abilities are not unbounded, there are certain limits within which we can apply the efforts.
- Thus, possible value of the effort e are located within some interval $[\underline{e}, \overline{e}]$.
- When we reward for good performance, the corresponding function $r(e)$ is convex.
- Thus the difference $r(e) - e$ is convex.
- Therefore, the selected value e_0 coincides either with $\underline{e}$ or with $\overline{e}$.
- We can dismiss the case $e_0 = \underline{e}$ when the reward is so small that it is not worth spending any effort.

15. Resulting Explanation (cont-d)

- So, we can conclude that $e_0 = \overline{e}$, i.e., the person selects the largest possible effort.
- This is exactly what we wanted to achieve.
- On the other hand, when we punish for mistakes, the corresponding function $r(e)$ is concave.
- Thus the difference $r(e) - e$ is concave.
- Therefore, the selected value e_0 is always located inside the interval $[\underline{e}, \overline{e}]$: $e_0 < \overline{e}$.
- Thus, the person will not select the largest possible effort – which is exactly what we wanted to avoid.
- This indeed explains why rewarding for good performance works better than punishment for mistakes.

16. Discussion

- What if we have both reward for good performance and punishment for mistakes, i.e.,

$$r(e) = \text{const} + c_+ \cdot \max(0, e - e_0) + c_- \cdot \min(0, e - e_0)?$$

- In this case, for $c_+ > c_-$, the function is still convex, i.e., we still get a very good performance.
- However, if $c_- > c_+$, the function becomes concave, and the performance suffers.
- Thus, to get good results, reward must be larger than punishment.

17. Discussion (cont-d)

- It is worth metioning that:
  - the optimal rewarding function

$$r(e) = r_+ + c_+ \cdot \max(0, e - e_0),$$

  - in effect, coincides (modulo linear transformations of input and output)
  - with the efficient "rectified linear" activation function $r(e) = \max(0, e)$ used in deep learning.
- So, not only people learn better when we use this function – computers learn better too!

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