# Why Quantiles Are a Good Description of Volatility in Economics: A Pedagogical Explanation

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# 1. What Is Volatility And Why It Is Important

- One of the main problems in economics is how to best invest money.

- Money can be invested in different financial instruments: stocks, bonds, derivatives, etc.

- For some of these instruments – e.g., for some bonds – we know exactly how much interest we will get.

- For other instruments, e.g., for stocks, we may gain a lot – or we may even lose, if the price goes down.

- In economics, this unpredictability of gain is known as *volatility*.

## 2. What Is Volatility (cont-d)

- Usually, the riskier instruments – i.e., instruments with higher volatility – provide better expected return.

- Indeed, otherwise:

  - why would someone invest in a risker instrument
  - if it is possible to invest if a less risky instrument with the same (or better) expected return.

# 3. How Can We Describe Volatility in Numerical Terms

- To make investment decisions, we need to be able to compare the risks of different instruments.

- For this, we need to be able to describe volatility in numerical terms.

- Originally, volatility was described in terms of the standard deviation of the return.

- However, standard deviation does not always adequately describe our intuitive idea of volatility:

  – sometimes instruments with higher standard deviations are perceived as less risky, and,

  – sometimes, vice versa, instruments with lower standard deviations are perceived as more risky.

- It turns out that a better description of intuitive volatility is provided by *quantiles* of the distribution.

# 4.   How Can We Describe Volatility (cont-d)

- A quantile is a value $x$ for which:

  - the probability $\mathrm{Prob}(r \leq x)$ that the return $r$ is smaller than or equal to $x$

  - is equal to some selected value $p$:

$$\mathrm{Prob}(r \leq x) = p.$$

- Here are some example of quantiles:

  - for $p = \dfrac{1}{2}$, we get a median;

  - for $p = \dfrac{1}{4}$ and $p = \dfrac{3}{4}$, we get quartiles, etc.

- Quantiles are usually defined in terms of the cumulative distribution function (cdf) $F(x) \stackrel{\text{def}}{=} \mathrm{Prob}(X \leq x)$.

- Namely, a quantile is the corresponding value $F^{-1}(p)$ of the inverse function.

## 5. How Can We Describe Volatility (cont-d)

- A natural question is:
  - why namely quantiles – and not any other characteristics of a probability distribution,
  - provide a description of volatility which is the closest to the intuitive understanding of volatility.

- In this talk, we provide an explanation.

# 6. Main Idea Behind Our Explanation

- In our explanation, we use the fact that the effect of money on a person's happiness is unusually nonlinear.

- This non-linearity is easy to explain:
  - if you have no money at all, and you gain a dollar, you are happy, but
  - if you already have $1,000 and you gain one more dollar, you will barely notice the difference.

- Psychological experiments show that this nonlinear dependence of happiness on money is:
  - somewhat different
  - for different people.

# 7. How Is This Related to the Intuitive Idea of Volatility?

- When people think of volatility:

  - they think in terms of changes in their happiness level,
  - and this level is related to money by some non-linear transformation.

- So, what we want is a characteristic that:

  - would not change
  - if we consider a different person, with a different non-linear function relating money and happiness.

- Let $X$ be a random variable that describes possible gains.

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# 8. Intuitive Idea of Volatility (cont-d)

- We want a volatility characteristic to have the following property:

- If the volatility characteristic measured in the money scale is $c(X)$, then:

  - for any monotonic re-scaling function $f(x)$, the volatility $c(Y)$ of a re-scaled variable $Y = f(X)$
  - should be equal to the same value $c(X)$ described in the new scale, i.e., we should have

$$c(f(X)) = f(c(X)).$$

# 9. This Natural Invariance Requirement Leads to Quantiles

- Let us show that this natural invariance requirement explains the appearance of quantiles.

- Indeed, it is known that if we have the cdf $F(x)$, then $Y = F(X)$ is uniformly distributed on $[0, 1]$.

- This fact is used to simulate probability distributions with a known cdf.

- Let $p$ denote the value of the desired characteristic $c(U)$ when $U$ is uniformly distributed on $[0, 1]$.

- Since $F(X) = U$, we have $c(F(X)) = p$.

- Thus, the above invariance requirement takes the form $c(F(X)) = F(c(X))$, i.e., the form $p = F(c(X))$.

# 10.    Invariance Leads to Quantiles (cont-d)

- By applying the inverse function $F^{-1}(z)$ to both sides of this equality, we conclude that $c(X) = F^{-1}(p)$.

- This is exactly the quantile value that we tried to explain.

- So, we have indeed explained why quantiles are the best description of intuitive notion of volatility.

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# 11. Acknowledgments

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