Why Micro-Funding? Why Small Businesses Are Important? Analysis Based on First Principles

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1. Economy of scale: well-known and well-studied phenomenon

- In economics, there is a known phenomenon of *economy of scale*, when a merger of two small companies helps lower the costs.
- The same phenomenon is known in all kinds of activities.
- For example, when researchers collaborate, they can usually achieve much more than when they work on their own or in small groups.
- One would expect that this effectiveness leads to the dominance of big companies in economics and big well-funded projects in science.
- In practice, however, there is a stable and significant proportion of small businesses.
- This shows that there is economic benefit in having small businesses in addition to big companies.

2. Micro-funding is useful

- Along the same lines, it has been empirically shown that:
  - the best way to stimulate economy
  - is to provide funding both to big and small businesses, i.e., to combine macro-funding and micro-funding.
- Similarly, when supporting science, the best effect is achieved:
  - when usual-size grants
  - are supplemented by micro-funding, i.e., by smaller-size grants.

3. How can we explain this phenomenon?

- In economics, in science sponsorship, and in other similar areas there are good explanations for this phenomenon.
- However, the current explanations are specific to each area, while the phenomenon is the same in all these areas.
- It is therefore desirable to look for a general explanation for this phenomenon.
- In this talk, we provide such a general explanation.

4. Let us formulate the problem in precise terms

- In all such situations:
  - we have a fixed amount of money, and
  - we want to find the best way to distribute this amount.
- Each distribution can be naturally described by a density function $f(m)$ for which:
  - the number of grants of sizes between m and Δm
  - is equal to $f(m) \cdot \Delta m$.
- In these terms, the question is: What is the optimal function $f(m)$?

5. What do we mean by optimal: towards a precise definition

- In the above formulation, we used the word "optimal".
- Usually, this means that we have an objective function – e.g.:
  - profit for a company,
  - research productivity for a group of researchers.
- We want to maximize the value of this function.
- However, in our case, we do not know the exact form of the objective function.
- All we know is that:
  - some distributions are more effective than others; we will denote this relation by $f(m) \succ g(m)$ – and
  - some distributions are of the same effectiveness as others; we will denote this relation by $f(m) \sim g(m)$.

6. These relations must be consistent

- If f is better than g and g is better than h, then we should be able to conclude that f is better than h.
- This leads to the following definition.

7. Definitions

By an optimality criterion *on a set* A*, we mean a pair of binary relations* $\langle \succ, \sim \rangle$ *that satisfy the following properties for all* $a, b, c \in A$*:*

- *if* $a \succ b$ *and* $b \succ c$*, then* $a \succ c$*;*
- *if* $a \succ b$ *and* $b \sim c$*, then* $a \succ c$*;*
- *if* $a \sim b$ *and* $b \succ c$*, then* $a \succ c$*;*
- *if* $a \sim b$ *and* $b \sim c$*, then* $a \sim c$*;*
- *if* $a \sim b$*, then* $b \sim a$*;*
- $a \sim a$*; and*
- *if* $a \succ b$ *then we cannot have* $a \sim b$*.*

We say that an element $a_0 \in A$ *is* optimal *with respect to the optimality criterion* $\langle \succ, \sim \rangle$ *if for every* $a \in A$*, we have either* $a_0 \succ a$ *or* $a_0 \sim a$*.*

8. Optimality criterion must be final

- It is reasonable to require that there is one and only one optimal alternative.
- Indeed, if there are no optimal alternatives, then this criterion is useless.
- So, there must be at least one optimal alternative.
- On the other hand:
  - if there are two or more alternatives of equal quality,
  - then we can use this non-uniqueness to optimize something else.
- For example, if a company has two alternatives with the same amount of expected profit, it is reasonable to select one with the lowest risk.
- This means, in effect, that in this case our original optimality criterion $\succ$ was not final.

9. Optimality criterion must be final (cont-d)

- We add additional criterion $\succ_a$ to formulate a new criterion $\succ_n$ for which $a \succ_n b$ if and only if:
  - either $a \succ b$ according to the original criterion,
  - or $a \sim b$ according to the original criterion and $a \succ_a b$ according to the additional criterion.
- If the new criterion still leads to several equally good optimal alternatives, this means that this new criterion is still not final.
- We can use this non-uniqueness to optimize something else.
- This process can continue until we reach the final optimality criterion, i.e., a criterion for which there is exactly one optimal alternative.
- This leads to the following definition.
- *We say that the optimality criterion is* final *if there exists exactly one alternative that is optimal with respect to this criterion.*

10. Optimality criterion should be scale-invariant

- Previously, we talked about general properties of optimality criteria.
- Let us now consider our case, when alternatives are non-negative functions $f(m)$ defined for positive values m.
- Each value m describes the amount of funding.
- The numerical value of funding depends on what units we choose for counting money: we can use dollars, Euros, Mexican pesos, etc.
- The choice of a monetary unit is a matter of convenience.
- It is therefore reasonable to require that the relative quality of different functions should not depend on what unit we use for counting.
- By definition of density, in the original units:
  - for each pair of values m and $\Delta m \ll m$,
  - we have $f(m) \cdot \Delta m$ folks who receive grants with amounts from m to $m + \Delta m$.

11. Optimality criterion should be scale-invariant (cont-d)

- If we replace a monetary unit with another one that is λ times smaller, then all the numerical values will be multiplied by λ:

$$m \mapsto m' = \lambda \cdot m \text{ and } \Delta m \mapsto \Delta m' = \lambda \cdot \Delta m.$$

- Let us denote by $f'(m')$ the probability density as described in the new units.
- In the new units, the same number of people $f(m) \cdot \Delta m$ is now equal to $f'(m') \cdot \Delta m'$: $f'(m') \cdot \Delta m' = f(m) \cdot \Delta m$.
- Here, $m = m'/\lambda$ and $\Delta m = \Delta m'/\lambda$; substituting these expressions for m and Δm into the above formula, we conclude that

$$f'(m') \cdot \Delta m' = f(m'/\lambda) \cdot \Delta m'/\lambda.$$

- Dividing both sides by $\Delta m'$, we conclude that the density in the new units has the form $f'(m') = f(m'/\lambda)/\lambda$.

12. Optimality criterion should be scale-invariant (cont-d)

- Thus, the dependence described, in the original unit, by a function $f(m)$ will now be described by a function $S_\lambda(f)$ for which

$$(S_\lambda(f))(m) \stackrel{\text{def}}{=} f(m/\lambda)/\lambda.$$

- So, the requirement that the relative quality not depend on the unit takes the following scale-invariance form:
  - if $f \succ g$, then $S_\lambda(f) \succ S_\lambda(g)$, and
  - if $f \sim g$, then $S_\lambda(f) \sim S_\lambda(g)$.
- Now, we are ready to formulate our main result.

13. Main result

- *For every scale-invariant final optimality criterion, the optimal function has the form* $f(m) = C/m$ *for some value* C.

14. Of course, our model is a simplification

- Of course, as usual in numerical analysis of real-life problems, our description is an idealization:
  - we assume that the monetary amount can take any real values,
  - but values which are too small or too large are not realistic.
- We cannot give a loan of 0.1 cents.
- And, for a different reason, we cannot give a loan of a thousand trillion dollars – no one has that much money.
- However, similar simplifications work well in many other applications.
- So we believe that our result well describes what is happening for realistic value m.
- This leads us to the following conclusion.

15. Main conclusion

- The optimal function – as described by the above proposition – is everywhere positive.
- So in the optimal arrangement, we should always have some grants with small m.
- This explains the ubiquity and effectiveness of micro-funding.

16. This leads to a new explanation for the ubiquity of Zipf's Law

- A specific form of the optimal function $f(m)$ is known as Zipf's Law.
- The law is ubiquitous.
- In particular, it describes the distribution of the companies by size – one of the phenomena that we are trying to explain.
- Thus, our result provide one more explanation of Zipf's law.

17. Proof

- We assumed that the optimality criterion is final.
- This means that there exists exactly one function that is optimal with respect to this criterion.
- Let us denote this optimal function by F.
- Let us first prove that the function F is itself scale-invariant, i.e., that for every $\lambda > 0$, we have $F = S_\lambda(F)$.
- Indeed, by definition of optimality, the fact that F is optimal means that for every $a \in A$ we have either $F \succ a$ or $F \sim a$.
- In particular, for every function a, we have

$$F \succ S_{1/\lambda}(a) \text{ or } F \sim S_{1/\lambda}(a).$$

- By scale-invariance, we can conclude that

$$S_\lambda(F) \succ S_\lambda(S_{1/\lambda}(a)) \text{ or } S_\lambda(F) \sim S_\lambda(S_{1/\lambda}(a)).$$

18. Proof (cont-d)

- One can easily see that $S_\lambda(S_{1/\lambda}(a)) = a$.
- Thus, for every $a \in A$, we have either $S_\lambda(F) \succ a$ or $S_\lambda(F) \sim a$.
- By definition of optimality, this means that the function $S_\lambda(F)$ is optimal.
- However, we assumed that the optimality criterion is final, which means that there is only one optimal function.
- Thus indeed $S_\lambda(F) = F$ for all $\lambda > 0$.
- By definition of the expression $S_\lambda(F)$, thus means that for every $m > 0$ and every $\lambda > 0$, we have $F(m/\lambda)/\lambda = F(m)$.
- In particular, for $\lambda = m$, we get $F(m) = C/m$, where we denoted $C \stackrel{\text{def}}{=} F(1)$.
- The proposition is proven.

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