How Do Degrees of Confidence Change with Time?

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1. Formulation of the Problem

- Situations change.
- As a result, our degrees of confidence in statements based on past experience decrease with time.
- How can we describe this decrease?
- If our original degree of confidence was a, what will be our degree of confidence $d_t(a)$ after time t?
- (It is clear that $d_t(a)$ should be increasing in a and decreasing in t, but there are many such functions.)

2. Our Idea

- Let $f_\&(a,b)$ be an "and"-operation, i.e., a function that transforms
  - degrees of confidence a and b in statements A and B
  - into an estimate for our degree of confidence in $A\,\&\,B$.
- There are two ways to estimate our degree of confidence in the statement $A\,\&\,B$ after time t:
  - we can apply the function d_t to both a and b, and then combine them into $f_\&(d_t(a),d_t(b))$,
  - or we can apply d_t directly to $f_\&(a,b)$, resulting in $d_t(f_\&(a,b))$.
- It is reasonable to require that these two expressions coincide:

$$f_\&(d_t(a),d_t(b)) = d_t(f_\&(a,b)).$$

3. Simplest Case

- If $f_\&(a, b) = a \cdot b$, then the above equality becomes $d_t(a \cdot b) = d_t(a) \cdot d_t(b)$.
- It is known that all monotonic solutions to this equation have the form $d_t(a) = a^{p(t)}$ for some $p(t)$.
- How to find the dependence on t?
- *Idea:* the decrease during time $t = t_1 + t_2$ can also be computed in two ways:
  - directly, as $a^{p(t_1+t_2)}$, or
  - by first considering decrease during t_1 $(a \to a' = a^{p(t_1)})$, and then a decrease during t_2:

$$a' \to (a')^{p(t_2)} = \left(a^{p(t_1)}\right)^{p(t_2)} = a^{p(t_1)\cdot p(t_2)}.$$

- It is reasonable to require that these two expressions coincide, i.e., that $p(t_1 + t_2) = p(t_1) \cdot p(t_2)$.

4. Simplest Case (cont-d)

- We concluded that

$$p(t_1 + t_2) = p(t_1) \cdot p(t_2).$$

- The only monotonic solutions to this equation are

$$p(t) = \exp(\alpha \cdot t).$$

- So, we get:

$$d_t(p) = a^{\exp(\alpha \cdot t)}.$$

5. Comment

- We get the formula:

$$d_t(p) = a^{\exp(\alpha \cdot t)}.$$

- For small t, we get:

$$d_t(p) \approx a + \alpha \cdot t \cdot a \cdot \ln(a).$$

- So, the above formula is related to entropy

$$-\sum a_i \cdot \ln(a_i).$$

6. General Case

- It is known that every "and"-operation can be approximated, with any accuracy,
  - by a Archimedean one,
  - i.e., by an operation of the type

$$f_\&(a.b) = g^{-1}(g(a) \cdot g(b)).$$

- Thus, for re-scaled values $a' = g(a)$, we have:

$$f_\&(a', b') = a' \cdot b'.$$

- Hence, $d_t(a') = (a')^{\exp(\alpha \cdot t)}$.
- In the original scale, we have the formula:

$$d_t(a) = g^{-1}\left((g(a))^{\exp(\alpha \cdot t)}\right).$$