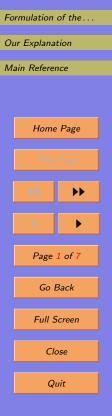
Towards a Natural Interval Interpretation of Pythagorean and Complex Degrees of Confidence

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1. Formulation of the Problem

- Often:
 - we only know the expert's degrees of confidence $a, b \in [0, 1]$ in statements A and B,
 - and we need to estimate the expert's degree of confidence in A & B.
- The algorithm $f_{\&}(a,b)$ providing the corresponding estimate is known as an "and"-operation, or a t-norm.
- One of the most frequently used "and"-operation is $\min(a, b)$.
- Similarly, one of the most frequently used "or"operation is

 $\max(a,b)$.

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2. Formulation of the Problem (cont-d)

- Often, it is difficult for an expert to describe his/her degree of certainty by a single number a.
- An expert is more comfortable describing it by range (interval) $[\underline{a}, \overline{a}]$ of possible values.
- An alternative way of describing this is as an *intuition-istic fuzzy degree*, i.e., a pair of values \underline{a} and $1 \overline{a}$.
- If we know:
 - intervals $[\underline{a}, \overline{a}]$ and $[\underline{b}, \overline{b}]$ corresponding to a and b,
 - then the range of possible degree of confidence in A & B is formed by values $\min(a, b)$ corresponding to all $a \in [\underline{a}, \overline{a}]$ and $b \in [\underline{b}, \overline{b}]$.
- Since $\min(a, b)$ is monotonic, this range has the form $[\min(a, b), \min(\overline{a}, \overline{b})].$
- Similarly, the range for $A \vee B$ is $[\max(\underline{a},\underline{b}), \max(\overline{a},\overline{b})]$.



3. Formulation of the Problem (cont-d)

- A recent paper describes extensions of the above definitions from $a, b \in [0, 1]$ to $a, b \in [-1, 1]$.
- These extensions are denoted by

 $[\operatorname{absmin}(\underline{a},\underline{b}),\operatorname{absmin}(\overline{a},\overline{b})]$ and $[\operatorname{absmax}(\underline{a},\underline{b}),\operatorname{absmax}(\overline{a},\overline{b})].$

- Here, absmin(a, b) = a if |a| < |b|.
- absmin(a, b) = b is |a| > |b|, and
- absmin(a,b)=-|a| if |a|=|b| and $a \neq b$.
- $\operatorname{absmax}(a, b) = a \text{ if } |a| > |b|.$
- absmax(a, b) = b if |a| < |b|, and
- $absmax(a, b) = |a| \text{ if } |a| = |b| \text{ and } a \neq b.$
- These operations have nice properties associativity, distributivity but what is their meaning?

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4. Our Explanation

- In addition to closed intervals, let us consider open and semi-open ones.
- An open end will be then denoted by the negative number:
 - for example, (0.3, 0.5] is denoted as [-0.3, 0.5], and
 - the interval (0.3, 0.5) is denoted as [-0.3, -0.5].
- By considering all possible cases, one can show that:
 - for two intervals $A = [\underline{a}, \overline{a}]$ and $B = [\underline{B}, \overline{B}]$,
 - the range of possible values

$$\{\min(a,b) : a \in A, b \in B\}$$

- is indeed equal to

$$[abs min(\underline{a}, \underline{b}), abs min(\overline{a}, \overline{b})].$$



5. Our Explanation (cont-d)

• For example, for A = (0.3, 0.7] = [-0.3, 0.7] and B = [0.2, 0.6) = [0.2, -0.6], we have

$${\min(a,b): a \in A, b \in B} = [0,2,0.6) = [0.2,-0.6].$$

- Here indeed:
 - we have absmin(-0.3, 0.2) = 0.2 and
 - we have absmin(0.7, -0.6) = -0.6.
- Similarly, the range of possible values of $\max(a, b)$ forms the interval

[abs
$$\max(\underline{a}, \underline{b})$$
, abs $\max(\overline{a}, \overline{b})$].

• Thus, we get a natural interval explanation of the extended operations.



6. Main Reference

• S. Dick, R. Yager, and O. Yazdanbakhsh, "On Pythagorean and complex fuzzy set operations", *IEEE Transactions on Fuzzy Systems*, 2016, Vol. 24, No. 5, pp. 1009–1021.

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