Towards a Natural Interval Interpretation of Pythagorean and Complex Degrees of Confidence

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1. Formulation of the Problem

- Often:
  - we only know the expert's degrees of confidence $a, b \in [0, 1]$ in statements A and B,
  - and we need to estimate the expert's degree of confidence in $A \,\&\, B$.
- The algorithm $f_{\&}(a, b)$ providing the corresponding estimate is known as an *"and"-operation*, or a *t-norm*.
- One of the most frequently used "and"-operation is

$$\min(a, b).$$

- Similarly, one of the most frequently used "or"-operation is

$$\max(a, b).$$

2. Formulation of the Problem (cont-d)

- Often, it is difficult for an expert to describe his/her degree of certainty by a single number a.
- An expert is more comfortable describing it by range (interval) $[\underline{a}, \overline{a}]$ of possible values.
- An alternative way of describing this is as an *intuitionistic fuzzy degree*, i.e., a pair of values $\underline{a}$ and $1 - \overline{a}$.
- If we know:
  - intervals $[\underline{a}, \overline{a}]$ and $[\underline{b}, \overline{b}]$ corresponding to a and b,
  - then the range of possible degree of confidence in $A \,\&\, B$ is formed by values $\min(a, b)$ corresponding to all $a \in [\underline{a}, \overline{a}]$ and $b \in [\underline{b}, \overline{b}]$.
- Since $\min(a, b)$ is monotonic, this range has the form $[\min(\underline{a}, \underline{b}), \min(\overline{a}, \overline{b})]$.
- Similarly, the range for $A \vee B$ is $[\max(\underline{a}, \underline{b}), \max(\overline{a}, \overline{b})]$.

3. Formulation of the Problem (cont-d)

- A recent paper describes extensions of the above definitions from $a, b \in [0,1]$ to $a, b \in [-1,1]$.
- These extensions are denoted by

$$[\text{absmin}(\underline{a}, \underline{b}), \text{absmin}(\overline{a}, \overline{b})] \text{ and } [\text{absmax}(\underline{a}, \underline{b}), \text{absmax}(\overline{a}, \overline{b})].$$

- Here, $\text{absmin}(a, b) = a$ if $|a| < |b|$.
- $\text{absmin}(a, b) = b$ is $|a| > |b|$, and
- absmin(a,b)=$-|a|$ if $|a| = |b|$ and $a \neq b$.
- $\text{absmax}(a, b) = a$ if $|a| > |b|$.
- $\text{absmax}(a, b) = b$ if $|a| < |b|$, and
- $\text{absmax}(a, b) = |a|$ if $|a| = |b|$ and $a \neq b$.
- These operations have nice properties – associativity, distributivity – but what is their meaning?

4. Our Explanation

- In addition to closed intervals, let us consider open and semi-open ones.
- An open end will be then denoted by the negative number:
  - for example, $(0.3, 0.5]$ is denoted as $[-0.3, 0.5]$, and
  - the interval $(0.3, 0.5)$ is denoted as $[-0.3, -0.5]$.
- By considering all possible cases, one can show that:
  - for two intervals $A = [\underline{a}, \overline{a}]$ and $B = [\underline{B}, \overline{B}]$,
  - the range of possible values

$$\{\min(a, b) : a \in A, b \in B\}$$

  - is indeed equal to

$$[\operatorname{abs}\min(\underline{a}, \underline{b}), \operatorname{abs}\min(\overline{a}, \overline{b})].$$

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5. Our Explanation (cont-d)

- For example, for $A = (0.3, 0.7] = [-0.3, 0.7]$ and $B = [0.2, 0.6) = [0.2, -0.6]$, we have

$$\{\min(a, b) : a \in A, b \in B\} = [0, 2, 0.6) = [0.2, -0.6].$$

- Here indeed:
  - we have $\mathrm{absmin}(-0.3, 0.2) = 0.2$ and
  - we have $\mathrm{absmin}(0.7, -0.6) = -0.6$.
- Similarly, the range of possible values of $\max(a, b)$ forms the interval

$$[\mathrm{abs}\max(\underline{a}, \underline{b}), \mathrm{abs}\max(\overline{a}, \overline{b})].$$

- Thus, we get a natural interval explanation of the extended operations.

6. Main Reference

- S. Dick, R. Yager, and O. Yazdanbakhsh, "On Pythagorean and complex fuzzy set operations", *IEEE Transactions on Fuzzy Systems*, 2016, Vol. 24, No. 5, pp. 1009–1021.