Safety Factors in Soil and Pavement Engineering: Theoretical Explanation of Empirical Data

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1. What Is a Safety Factor

- Models are approximations to reality.
- To describe a complex real-life process by a feasible model:
  - we find the most important factors affecting the process and
  - we model them.
- The ignored factors are smaller than the factors that we take into account; however:
  - they still need to be taken into account
  - if we want to provide guaranteed bounds for the desired quantities.
- To take these small factors into account, engineers multiply the results of the model by a constant.
- This constant is known as the *safety factor*.

2. Safety Factors in Soil and Pavement Engineering: Empirical Data

- In many applications, a safety factor is 2 or smaller.
- However, in soil and pavement engineering, the situation ia different.
- Researchers compared:
  - the resilient modulus predicted by the corresponding model and
  - the modulus measured by Light Weight Deflectometer.
- This comparison showed that:
  - to provide guaranteed bounds,
  - we need a safety factor of 4.
- How can we explain this?

3. Explaining the Safety Factor of 2: Reminder

- Let Δ be the model's estimate.
- When designing the model, we did not take into account some factors.
- Let's denote the effect of the largest of these factors by Δ_1.
- The factors that we ignored are smaller than the one we took into account, so $\Delta_1 < \Delta$, i.e., $\Delta_1 \in [0, \Delta]$.
- We do not have any reason to assume that any value from the interval $[0, \Delta]$ is more frequent than others.
- Thus, it makes sense to assume that Δ_1 is uniformly distributed on $[0, \Delta]$.
- Then, the average value of Δ_1 is $\Delta/2$.
- The next smallest factor Δ_2 is smaller than Δ_1.

4. Explaining the Safety Factor of 2 (cont-d)

- The same arguments shows that its average value of Δ_2 is $\Delta_1/2$, i.e., $\Delta_2 = 2^{-2} \cdot \Delta$.
- Similarly, $\Delta_k = 2^{-k} \cdot \Delta$.
- Hence the overall estimate is

$$\Delta + \Delta_1 + \ldots = \Delta + 2^{-1} \cdot \Delta + \ldots + 2^{-k} \cdot \Delta + \ldots = 2\Delta.$$

5. A Similar Explanation for the Safety Factor of 4

- Empirical data shows that for soil and pavement engineering, 2 is not enough.
- This means that Δ_1 should be larger than our estimate $\Delta/2$: $\Delta_1 \in [\Delta/2, \Delta]$.
- In this case, the average value from this interval is

$$\Delta_1 = (3/4) \cdot \Delta.$$

- Similarly, we get $\Delta_2 = (3/4)^2 \cdot \Delta$, $\Delta_k = (3/4)^k \cdot \Delta$ and thus,

$$\Delta + \Delta_1 + \ldots + \Delta_k + \ldots = \Delta \cdot (1 + 3/4 + \ldots + (3/4)^k + \ldots) =$$

$$\Delta/(1 - 3/4) = 4\Delta.$$

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7. Main Reference

- M. Mazari, E. Navarro, I. Abdallah, and S. Nazarian, "Comparison of numerical and experimental responses of pavement systems using various resilient modulus models", *Soils and Foundations*, 2014, Vol. 54, No. 1, pp. 36–44.