Computing with Words – When Results Do Not Depend on the Selection of the Membership Function

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1. Formulation of the Problem

- Often, we need to transform natural-language expert knowledge into computer-understandable numerical form.
- One of the most successful ways to do it is to use fuzzy logic.
- In fuzzy logic, each imprecise property like "small" is described by a *membership function* that assigns,
  - to each possible value x,
  - a degree $\mu(x)$ to which x is, e.g., small.
- Problem: membership functions are subjective.
- It is therefore desirable to look for cases when the results do not depend on this subjective choice.

2. Continuity: Known Example

- Intuitively, continuity means that if x' is close to x, then $y' = f(x')$ should be close to $y = f(x)$.
- In other words, if $x' - x$ is small, then $f(x') - f(x)$ should be small.
- Thus, the degree $\mu^y_{\text{small}}(f(x') - f(x))$ cannot be smaller that $\mu^x_{\text{small}}(x' - x)$.
- The quantities x and y may differ by scale, so $\mu^y_{\text{small}}(z) = \mu^x_{\text{small}}(K \cdot z)$.
- Thus, $\mu^x_{\text{small}}(K \cdot (f(x') - f(x)) \geq \mu^x_{\text{small}}(x' - x)$.
- Hence $K \cdot |f(x') - f(x)| \leq |x - x'|$ and $|f(x') - f(x)| \leq K^{-1} \cdot |x' - x|$.
- Thus, the common sense continuity leads to the Lipschitz condition.

3. First New Example

- What if we have a relation between x and y and not a function?
- In this case, continuity still implies that $f(x)$ is a function.

4. Second New Example

- What is the dependence of y on x and x on y are both continuous?
- Then, we have $|f(x') - f(x)| \le K^{-1} \cdot |x' - x|$ and $|x' - x| \le K \cdot |f(x') - f(x)|$.
- Hence $|f(x') - f(x)| = K^{-1} \cdot |x' - x|$ for all x and x'.
- One can prove that this is only possible when $f(x)$ is linear.
- This may explain the ubiquity of linear dependencies.

5. Third New Example

- What if $f(x)$ is growing?
- Intuitively, it means that if $x' \gg x$, then $f(x') \gg f(x)$.
- For any membership function for "much larger", we get $f(x') - f(x) \ge K \cdot (x' - x)$ for $x' > x$.
- This means, in effect, that $f'(x) \ge K$ for some $K > 0$.