Why People Overestimate Small Probabilities?

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1. Formulation of the problem

- It is known that people routinely overestimate small probabilities when making decisions.
- They overestimate the probability of rare side effects – and thus, refuse to take important vaccinations.
- Experiments performed by the Nobelist Daniel Kahneman and his team show that indeed, most people overestimate small probabilities.
- This is a fact, but how can we explain this fact from the biological viewpoint?
- At first glance, the more adequately we understand the situation, the more adequate decision we can make.
- So why did evolution preserve this clearly biased perception of small probabilities?

2. How do we know probabilities?

- Probabilities are estimates based on our experience.
- If we saw some event n times out of N, then we estimate the probability as the ratio n/N.
- But of course, this is only an approximate estimate.
- If we flip a perfectly symmetric coin 10 times:
  - we may get $n = 5$ heads,
  - but we may also get 6 or 4 or 7.

3. Which outcomes are possible?

- If an event has probability p, how may times out of N can it occur?
- If the actual probability is p, then out of N tries:
  - the event happens on average in $\mu \stackrel{\text{def}}{=} p \cdot N$ times, and
  - the variance of number of events is equal to $\sigma^2 = N \cdot p \cdot (1 - p)$.
- For small p, we have $1 - p \approx 1$, so $\sigma^2 \approx \mu$ and thus, $\mu \approx \sigma^2$.
- Usually:
  - if we have a distribution with a known mean and standard deviation,
  - we conclude – with high confidence – that the actual value is somewhere between $\mu - k \cdot \sigma$ and $\mu + k \cdot \sigma$.
- Here, $k = 2, 3, 6, \ldots$ depending on the desired level of confidence.

4. So what can we conclude about the probability?

- Suppose that some event occurred n time out of N.
- So, the only information that we can conclude about its probability p is that $\mu - k\cdot\sigma \le n \le \mu + k\cdot\sigma$.
- Equivalently, $\sigma^2 - k\cdot\sigma \le n \le \sigma^2 + k\cdot\sigma$, where $p = \sigma^2/N$.
- By using the known properties of quadratic equations and inequalities, we conclude that

$$\frac{\sqrt{k^2+4n}-k}{2} \le \sigma \le \frac{\sqrt{k^2+4n}+k}{2}, \text{ so}$$

$$\underline{p} \stackrel{\text{def}}{=} \frac{2n+k^2-k\cdot\sqrt{k^2+4n}}{2N} \le p \le \overline{p} \stackrel{\text{def}}{=} \frac{2n+k^2+k\cdot\sqrt{k^2+4n}}{2N}.$$

5. Which probability value should we select?

- We know that

$$\underline{p} \stackrel{\text{def}}{=} \frac{2n + k^2 - k \cdot \sqrt{k^2 + 4n}}{2N} \le p \le \overline{p} \stackrel{\text{def}}{=} \frac{2n + k^2 + k \cdot \sqrt{k^2 + 4n}}{2N}.$$

- We have no reason to consider one of the values from the interval $[\underline{p}, \overline{p}]$ as more probable.
- So, it makes sense to consider all these values equally possible.
- In this case, a natural idea is to select the average of these values, i.e., the midpoint

$$\frac{\underline{p} + \overline{p}}{2} = \frac{n}{N} + \frac{k^2}{2N}.$$

- This value is always larger than the frequency n/N – which is the usual (and unbiased) estimate of the actual probability.
- This provides a possible explanation of why we, in general, overestimate the values of small probabilities.

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