Fault Detection in a Smart Electric Grid: Geometric Analysis

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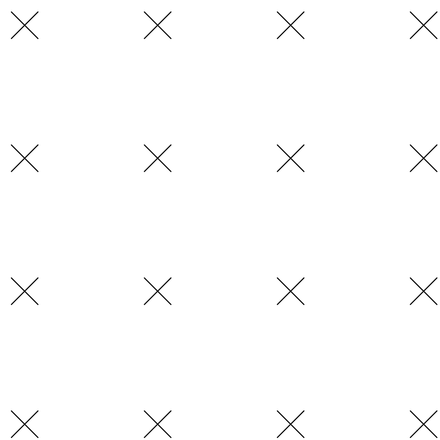
1. What is a smart electric grid

- The main idea is to set up a lattice of sensors that would monitor the electric grid.
- Based on the measurement results provided by the sensors:
  - we would get a good picture of the current state of the grid, and
  - we would be able to effectively control it.



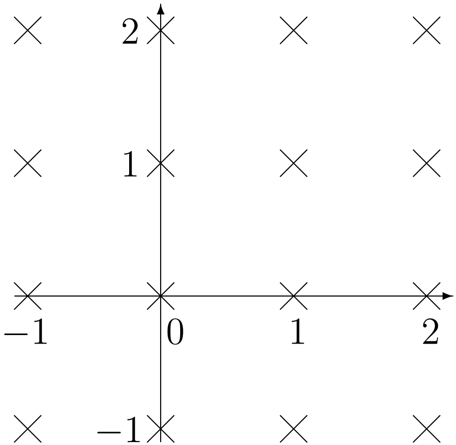
2. How the grid of sensor can detect faults

- Each sensor measures characteristics of the electric current at its location.
- Each fault affects all the sensors, some more, some less.
- By observing the changes in the sensor signals, we can detect the existence of the fault.
- We can also get some information of the fault's location.
- Sensors closer to the fault's location will detect a stronger change in their measurements results than sensors which are further away.
- Thus, by comparing the measurement results of the two sensors, we can decide whether the fault is:
  - closer to the first sensor or
  - closer to the second sensor.

3. Let us describe this situation in precise terms

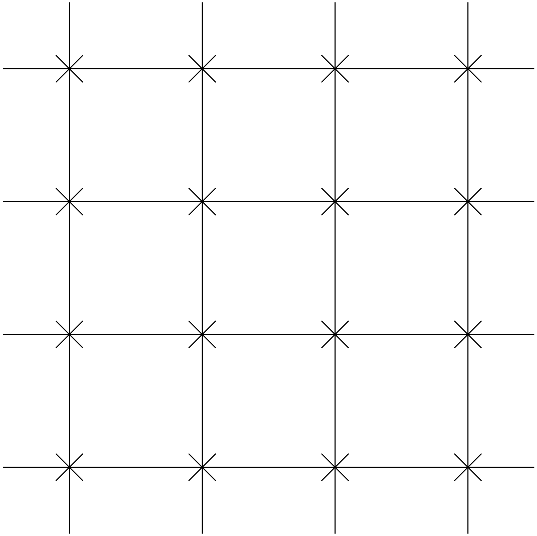
- Let us consider the case when the sensors form a (potentially infinite) rectangular lattice.
- For simplicity of analysis, let us select a coordinate system in which:
  - the location of one the sensors is the starting point $(0, 0)$, and
  - the distance between the closest sensors is used as a measuring unit.
- In this coordinate system, sensors are located at all the points (a, b) with integer coordinates.

4. Let us describe this situation in precise terms (cont-d)



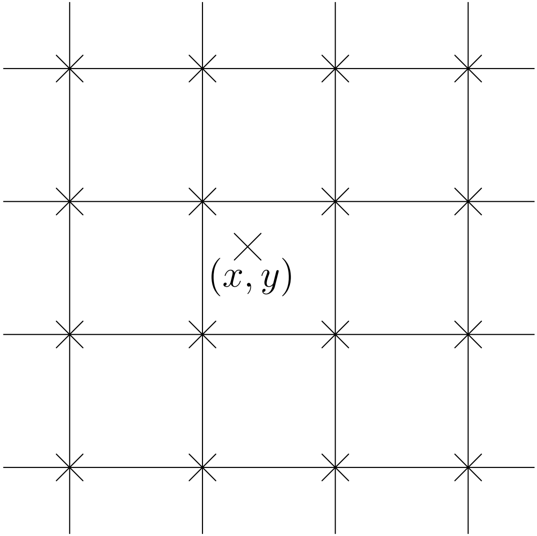
5. Let us describe this situation in precise terms (cont-d)

- These sensors divide the plane into squares $[a, a+1] \times [b, b+1]$.



6. Let us describe this situation in precise terms (cont-d)

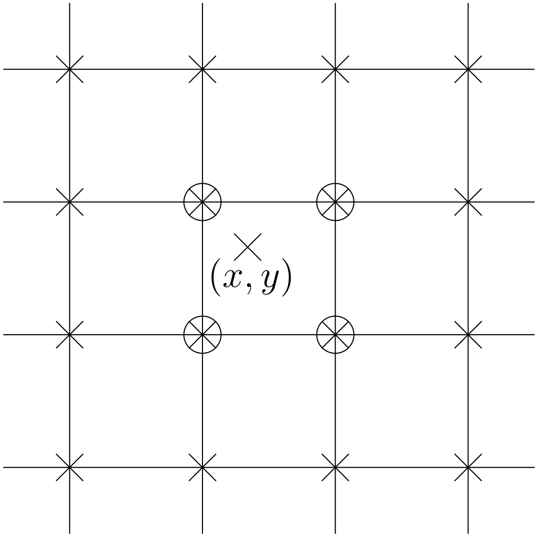
- Each spatial location (x, y) is in one of these squares.



7. Let us describe this situation in precise terms (cont-d)

- One can easily check that:
  - for each spatial location within a square,
  - the vertices (a, b), $(a, b+1)$, $(a+1, b)$, and $(a+1, b+1)$ of this square are the closest grid points.
- Thus:
  - by finding the 4 sensors at which the disturbance signal is the strongest,
  - we can find the square that contains the location of the fault.

8. Let us describe this situation in precise terms (cont-d)



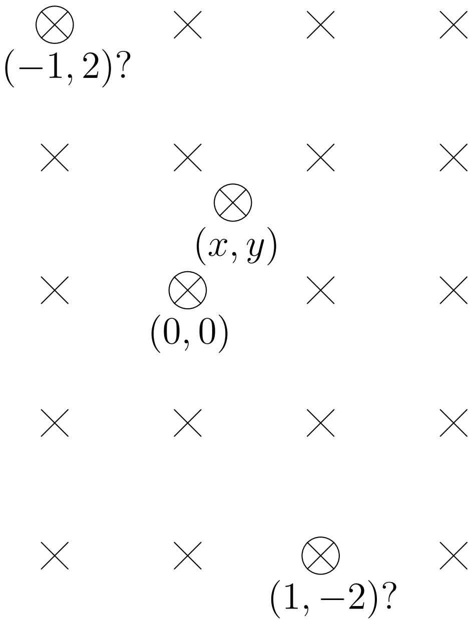
9. Research question

- Can we determine the location of the fault more accurately than "somewhere in the square"?

10. Our answer

- We show that, in principle:
  - by using the lattice of sensors,
  - we can locate the fault with any desired accuracy.
- Indeed, without losing generality, let us assume that the square containing the fault is the square $[0,1]\times[0,1]$.
- In other words, we know that the coordinates (x,y) of the fault satisfy the inequalities $0\le x\le 1$ and $0\le y\le 1$.
- For each pair of positive integers (p,q), we can check whether
  - the sensor at $(p,-q)$ gets a stronger signal than
  - the sensor at $(-p,q)$.

11. Our answer (cont-d)

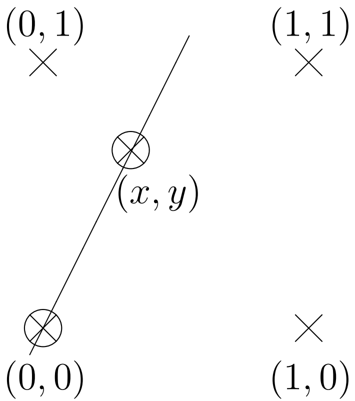


12. Our answer (cont-d)

- The first sensor's signal is stronger if and only if:
  - the squared distance $d^2(f, s_1) = (x - p)^2 + (y - (-q))^2$ between the fault f and the first sensor s_1 is smaller than
  - the squared distance $d^2(f, s_2) = (x - (-p))^2 + (y - q)^2$ to the second sensor.
- One can check that $d^2(f, s_1) < d^2(f, s_2)$ if and only if $q \cdot y < p \cdot x$, i.e., if and only if $y/x < p/q$.
- A real number can be uniquely determined if we know:
  - which rational numbers p/q are smaller than this number and
  - which are larger.
- Thus:
  - by comparing signals from different sensors,
  - we can determine the ratio $r \stackrel{\text{def}}{=} y/x$ with any given accuracy.

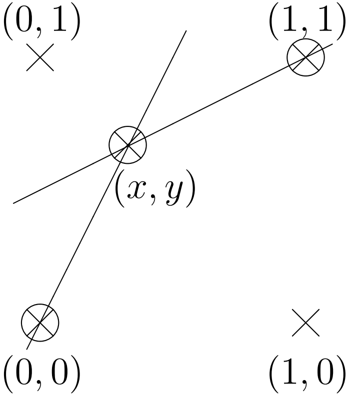
13. Our answer (cont-d)

- Hence, we can determine the line $y = r \cdot x$ going through $(0, 0)$ that contains the fault.



14. Our answer (cont-d)

- Similarly, we can find a straight line going through the point $(1, 1)$ that contains the fault.
- Thus:
  - the fault's location can be uniquely determined
  - as the intersection of these two straight lines.



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