Hunting Habits of Predatory Birds: Theoretical Explanation of an Empirical Formula

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1. Formulation of the Problem

- Predatory birds are an important part of an ecosystem.
- Like all predators, they help maintain the healthy balance in nature.
- This balance is very delicate, unintended human interference can disrupt it.
- To avoid such disruption, it is important to study the hunting behavior of predatory birds.
- This behavior is cyclic.
- Most predatory birds like owls spend some time waiting for the prey, and then either attack or jump to a new location.
- For the same bird, waiting time w changes randomly from one cycle to another.

2. Formulation of the Problem (cont-d)

- Researchers recently found how the probability $f(t) \stackrel{\text{def}}{=} \text{Prob}(w \geq t)$ that the waiting time is $\geq t$ depends on t: $f(t) \approx A \cdot t^{-a}$.
- How can we explain this empirical observation?

3. We Need a Family of Functions

- Some birds tend to wait longer, some tend to wait less.
- So, we cannot have a single formula that would cover all the birds of the same species.
- We need a family of functions $f(t)$.
- The simplest family if when we fix some function $F(t)$ and consider all possible functions of the type $C \cdot F(t)$.
- What family should we choose?

4. Invariance

- The numerical value of waiting time depends on the selection of the measuring unit.
- If we replace the original measuring unit with the one which is λ times smaller, all numerical values multiply by λ: $t \mapsto \lambda \cdot t$.
- It looks like there is no preferable measuring unit.
- So, it makes sense to assume that the family $\{C \cdot F(t)\}_C$ should remain the same if we change the measuring unit.
- This implies, in particular, that for every $\lambda > 0$, the function $F(\lambda \cdot t)$ should belong to the same family.
- Thus, for every $\lambda > 0$, there exists a constant C depending on λ for which $F(\lambda \cdot t) = C(\lambda) \cdot F(t)$.

5. Invariance

- It is known that every measurable solution to the functional equation $F(\lambda \cdot t) = C(\lambda) \cdot F(t)$ has the form $F(t) = A \cdot t^a$.
- This is exactly the empirical probability distribution – it is only one which does not depend on the selection of the measuring unit for time.

6. How to Prove the Result about the Functional Equation

- This result is easy to prove when the function $F(t)$ is differentiable.
- Suppose that $F(\lambda \cdot t) = C(\lambda) \cdot F(t)$.
- If we differentiate both sides with respect to λ, we get
$$t \cdot F'(\lambda \cdot t) = C'(\lambda) \cdot F(t).$$
- In particular, for $\lambda = 1$, we get $t \cdot F'(t) = a \cdot F(t)$, where $a \stackrel{\text{def}}{=} C'(1)$, so $t \cdot \dfrac{dF}{dt} = a \cdot F$.
- We can separate the variables if we multiply both sides by $\dfrac{dt}{t \cdot F}$, then we get $\dfrac{dF}{F} = a \cdot \dfrac{dt}{t}$.
- Integrating both sides of this equality, we get $\ln(F) = a \cdot \ln(t) + C$.
- By applying $\exp(x)$ to both sides, we get
$$F(t) = \exp(a \cdot \ln(t) + C) = A \cdot t^a, \text{ where } A \stackrel{\text{def}}{=} e^C.$$

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