# Resource Allocation for Multi-Tasking Optimization: Theoretical Explanation of an Empirical Formula

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# 1. Formulation of the Problem

- In many practical situations, we have several possible objective functions.
- For example, when we design a plant:
  - we can look for the cheapest design,
  - we can look for the most durable design,
  - we can look for the most environmentally friendly design, etc.
- To make a decision, it is desirable to find designs which are optimal with respect to all these criteria.
- This way:
  - if we find, e.g., that the most environmentally friendly design is close to the cheapest one,
  - we can add a little more money and make the design environmentally friendly.

# 2. Formulation of the Problem (cont-d)

- On the other hand:
  - if these two designs are too far away,
  - we can apply for an environment-related grant to make the design environment friendly.
- Optimizing different functions on the same domain involves several common domain-specific computational modules.
- Thus, it makes sense to perform all these optimization tasks on the same computer.
- In this case, the important question is how to distribute resources between different tasks.
- When one task is close to completion, it does not require many resources, while other tasks may require a lot.

# 3. Formulation of the Problem (cont-d)

- At each time period, we need to distribute, e.g., the available computation time $\Delta T$ between tasks, i.e., find value $\Delta t_k$ for which

$$\Delta t_1 + \Delta t_2 + \ldots = \Delta T.$$

- We should select the values $\Delta t_k$ for which the overall gain is the largest

$$\sum_k g_k(t_k + \Delta t_k) \to \max\,.$$

- Here, $t_k$ is the time already spent on this task.
- We do not know exactly how each gain $g_k$ will change with time.
- A natural idea is:
  - to select a family of functions $g(t)$ depending on a few parameters;
  - for each task, to find parameters that lead to the best fit;
  - then use these parameters to predict the value $g_k(t_k + \Delta t_k)$.

## 4. Formulation of the Problem (cont-d)

- Empirical fact is that among 2-parametric families, the best results are achieved for $g(t) = a \cdot \ln(t) + A$.
- How can we explain this empirical fact?

# 5. Our Explanation

- The numerical value of each physical quantity depends:
  - on the selection of the measuring unit, and
  - on the selection of the starting point.
- If we replace the original measuring unit with the one which is $\lambda$ times smaller, all numerical values multiply by $\lambda$: $x \mapsto \lambda \cdot x$.
- If we select a new starting point which is $x_0$ units smaller, then we get $x \mapsto x + x_0$.
- In many cases, there is no preferable measuring unit.
- In this case, it makes sense to assume that the formulas should remain the same if we change the measuring unit.
- In our case, there is a clear starting point for time: the moment when computations started.
- However, there is no preferred measuring unit.

## 6. Our Explanation (cont-d)

- Thus, it is reasonable to require that if we change $t$ to $\lambda \cdot t$, we will get the same resource allocation.
- This does not necessarily mean that $g(\lambda \cdot t) = g(t)$, since functions $g_k(t)$ and $g_k(t) + \text{const}$ lead to the same resource allocation.
- Thus, we require that for every $\lambda > 0$, we have $g(\lambda \cdot t) = g(t) + c$ for some constant $c$ depending on $\lambda$.
- It is known that every measurable solution to this functional equation has the form $g(t) = a \cdot \ln(t) + A$.
- This explains the empirical fact.
- Dependencies $g(t) = a \cdot \ln(t) + A$ are the only ones that do not depend on the choice of a measuring unit for time.

## 7. How to Prove the Result about the Functional Equation

- This result is easy to prove when the function $f(X)$ is differentiable.
- Suppose that $g(\lambda \cdot X) = g(X) + c(\lambda)$.
- If we differentiate both sides with respect to $\lambda$, we get

$$X \cdot g'(\lambda \cdot X) = c'(\lambda).$$

- In particular, for $\lambda = 1$, we get $X \cdot g'(X) = a$, where $a \stackrel{\text{def}}{=} c'(1)$, so

$$X \cdot \frac{dg}{dX} = a.$$

- We can separate the variables if we multiply both sides by $\dfrac{dX}{X}$, then we get $dg = a \cdot \dfrac{dX}{X}$.
- Integrating both sides of this equality, we get $g(X) = a \cdot \ln(X) + C$.

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