Dielectric Barrier Discharge (DBD) Thrusters – Aerospace Engines of the Future: Invariance-Based Analysis

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1. How to Fly on Mars: Dielectric Barrier Discharge (DBD) Thrusters

- A large amount of information about Earth comes from air-based observations.
- It is therefore desirable to have similar studies of planets with atmosphere.
- One of the problems is that on Earth, flying devices use fuel-based engines.
- On other planets, we do not have ready sources of fuel, and bringing fuel from Earth is too expensive.
- The main source of energy for planetary missions is electricity.
- Electricity can be generated by solar batteries and/or by radioactive energy sources.
- It is therefore desirable to use electricity to power the flying devices.

2. How to Fly on Mars: DBD Thrusters (cont-d)

- A natural idea is to use electrostatic forces between two electrodes.
- When the voltage is high, an electric arc appears – as in a lightning.
- The arc means that atmospheric atoms are ionized, into negatively charges electrons and positively changed ions.
- Ions move towards the negative electrode, while electrons move towards the positive electrode.
- Since the ions move towards the negative electrode, the density near the negative electrode becomes smaller than nearby.
- Thus, the atmospheric gases are sucked into this area.
- These gases also become ionized, so they also move towards the negative electrode.

3. How to Fly on Mars: DBD Thrusters (cont-d)

- The mass of ions is much larger than the mass of electrons, so the ion flow produces momentum and thus, thrust.
- This is the main idea behind what is called Dielectric Barrier Discharge Thrusters.

4. DBD Thrusters Are Useful on Earth Too

- While DBD thrusters were originally designed at NASA for planetary research, they are useful on Earth too.
- They do not have moving parts, so they are durable and reliable.
- They do not burn fuel, so they do not pollute the environment.
- They have a higher efficiency, for the following reason.
- In fuel-using flying devices, energy is wasted on two stages:
  - when fuel is burning – a large part of energy goes into useless heat, and
  - when turbines are used – a part of energy is spent on friction.
- In the electric device, there are only one stage, so fewer energy is wasted.

5. What Electric Field E Should We Select

- For a given design with given E, the efficiency of a thruster changes with atmospheric pressure p.
- When the pressure is too low, we do not have enough ions to generate thrust.
- On the other hand, when the pressure is too high, the air resistance becomes too strong.
- When the atmosphere is very dense, moving through it becomes practically impossible.
- For each E, there is an optimal pressure at which the thruster is the most efficient.
- So, for each value of the atmospheric pressure, we should select this optimal E.
- The atmospheric pressure decreases with height, so we should thus have E changing with height.

6. What Electric Field E Should We Select (cont-d)

- To find the optimal E, we need to know how the thrust F depends on p.
- At present, we only have an approximate semi-empirical formula $F(p) = c \cdot p \cdot \exp(a \cdot p)$ based on a simplified model.
- It is therefore desirable to provide a theoretical explanation for this formula.
- Another issue is that this formula provides a rather crude approximation to the data.
- It is desirable to come up with more accurate formulas.

7. We Should Select a Family of Functions $F(p)$

- For different designs, for different E, we have, in general, different dependencies $F(p)$.
- So, we cannot have a single function $F(p)$, we should select a family of functions.
- A natural way to describe a family of function is to select functions $e_1(p), \ldots, e_n(p)$, and consider all possible linear combinations

$$C_1 \cdot e_1(p) + \ldots + C_n \cdot e_n(p).$$

- For example, when $e_1(p) = 1$, $e_2(p) = p$, $e_3(p) = p^2$, we get a family of polynomials.
- Which family should we select?

8. Shifts and Shift-Invariance

- An important feature of pressure is that its effects, in some sense, do not depend on the selection of the starting point.
- For example, when there are no cars on the road:
  - we say that the pressure on the pavement is 0,
  - while in reality, there is always a strong atmospheric pressure.
- We can select different starting points, e.g., we can take 0 as vacuum or 0 as atmospheric pressure.
- If we replace the original starting point with the one which is p_0 units smaller, all numerical values are shifted: $p \mapsto p + p_0$.
- The selection of a starting point is rather arbitrary.
- So, it makes sense to select an approximating family that does not change under such shifts.
- Shift-invariance means that for each function $e_i(p)$, its shift $e_i(p+p_0)$ belongs to the same family.

9. Shifts and Shift-Invariance (cont-d)

- So, for some coefficients $C_{ij}(p_0)$, we get:

$$e_i(p+p_0)=C_{i1}(p_0)\cdot e_1(p)+\ldots+C_{in}(p_0)\cdot e_n(p).$$

- If we differentiate both sides with respect to p_0, we get

$$e_i'(p+p_0)=C_{i1}'(p_0)\cdot e_1(p)+\ldots+C_{in}'(p_0)\cdot e_n(p).$$

- In particular, for $p_0=0$, we get

$$e_i'(p)=c_{i1}\cdot e_1(p)+\ldots+c_{in}\cdot e_n(p).$$

- Here, we denoted $c_{ij}\stackrel{\text{def}}{=}C_{ij}'(0)$.
- So, we get a system of linear differential equations with constant coefficients.

10. Shifts and Shift-Invariance (cont-d)

- The functions $e_i(p)$ satisfies a system of linear differential equations with constant coefficients: $e_i'(p) = c_{i1} \cdot e_1(p) + \ldots + c_{in} \cdot e_n(p)$.
- It is known that all solutions to such systems are linear combinations of functions $t^m \cdot \exp(k \cdot t)$.
- Here, k is an eigenvalue of the matrix c_{ij}, and m is a non-negative integer which is smaller than k's multiplicity.
- The simplest case if when $n = 1$.
- In this case, we get functions $F(p) = C_1 \cdot \exp(k \cdot p)$.
- This does not satisfy the condition $F(0) = 0$.
- To satisfy this condition, we need to take $n \geq 2$.

11. Shifts and Shift-Invariance (cont-d)

- The simplest such case is $n = 2$.
- If we have an eigenvalue of multiplicity 2, we get

$$F(p) = C_1 \cdot \exp(k \cdot p) + C_2 \cdot p \cdot \exp(k \cdot p).$$

- The condition $F(0) = 0$ implies that $C_1 = 0$, so we have exactly the semi-empirical formula.
- When eigenvalues are complex-valued $k = a \pm b \cdot \mathrm{i}$, then

$$F(p) = \exp(a \cdot p) \cdot (C_1 \cdot \cos(b \cdot p) + C_1 \cdot \sin(b \cdot p)).$$

- This expression is negative for some p, while the force is always non-negative.

12. Shifts and Shift-Invariance (cont-d)

- When eigenvalues $k_1 < k_2$ are real and different, we get

$$F(p) = C_1 \cdot \exp(k_1 \cdot p) + C_2 \cdot \exp(k_2 \cdot p).$$

- The condition $F(0) = 0$ implies $C_2 = -C_1$, so

$$F(p) = C_1 \cdot (\exp(k_1 \cdot p) - \exp(k_2 \cdot p)).$$

- In the limit $k_2 \to k_1$, we get the expression $F(p) = c \cdot p \cdot \exp(k \cdot p)$.
- In general, we get a 3-parametric family that may lead to a better description of experimental data.
- If this will be not accurate enough, we can use shift-invariant families with $n = 3, 4, \ldots$

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