

# What If an Interval Is Given by an Expert Who Is Not Sure?

Arim S. Martin Del Campo, Antonio I. Rosales, and Vladik Kreinovich

Department of Computer Science, University of Texas at El Paso  
500 W. University, El Paso, Texas 79968, USA  
asmartindel@miners.utep.edu, airosales3@miners.utep.edu, vladik@utep.edu

## 1. Formulation of the problems

- In many real-life situations, we ask experts to estimate the probability  $p$  of a certain event.
- Example: the probability of an earthquake of certain strength.
- Expert estimates are, of course, approximate, experts understand this.
- So they can usually provide not only the numerical estimate  $\tilde{p}$ , but also some information about the accuracy of their estimate.
- This information often comes as a bound  $\Delta$  on the absolute value of the estimation error  $\Delta p \stackrel{\text{def}}{=} \tilde{p} - p$ .
- For example, an expert may say that the probability is  $80\% \pm 10\%$ .

## 2. Formulation of the problems (cont-d)

- In this case, what the expert provides is an *interval* of possible probabilities:
  - in the above example, the interval  $[70, 90]$ , and
  - in general, the interval  $[\underline{p}, \bar{p}] = [\tilde{p} - \Delta, \tilde{p} + \Delta]$ .
- Here are the two related problems:
  1. Sometimes, in addition to the interval, the expert also provides a degree of confidence  $c$  in his/her estimate – e.g., 80%.
    - \* How can we combine this information with the interval?
  2. And what if the expert cannot provide the degree of confidence? How can we estimate this degree?

### 3. Possible solution to the first problem

- With confidence  $c$ , the expert believes that the actual probability is in the interval  $[\underline{p}, \bar{p}]$ .
- This means that with the remaining probability  $1 - c$ :
  - the expert does not have any information,
  - i.e., the probability can be anywhere in  $[0, 1]$ .
- The overall expected probability is thus  $c \cdot p_1 + (1 - c) \cdot p_2$ , where  $p_1 \in [\underline{p}, \bar{p}]$  and  $p_2 \in [0, 1]$ .
- According to interval computations, the set of all such values is

$$c \cdot [\underline{p}, \bar{p}] + (1 - c) \cdot [0, 1] = [c \cdot \underline{p}, 1 - c \cdot (1 - \bar{p})].$$

## 4. Possible solution to the second problem

- Suppose that we have an even more reliable (but more difficult-to-consult) expert whose opinion we treat as accurate.
- On several examples  $k = 1, \dots, n$ , we ask the opinions of both experts, and get:
  - intervals  $[\underline{p}_k, \bar{p}_k]$  from the expert whom we are evaluating, and
  - intervals  $[\underline{r}_k, \bar{r}_k]$  from the more reliable expert.
- We then expect that  $c \cdot \underline{p}_k \approx \underline{r}_k$  and  $1 - c \cdot (1 - \bar{p}_k) \approx \bar{r}_k$ .
- Thus, we can find  $c$  by applying, e.g., the Least Squares approach to this system of approximate equalities.
- This means that we find  $c$  for which

$$\sum_{k=1}^n \left[ (c \cdot \underline{p}_k - \underline{r}_k)^2 + (1 - c \cdot (1 - \bar{p}_k) - \bar{r}_k)^2 \right] \rightarrow \min_c.$$

## 5. Interval computations: general references

- L. Jaulin, M. Kiefer, O. Didrit, and E. Walter, *Applied Interval Analysis, with Examples in Parameter and State Estimation, Robust Control, and Robotics*, Springer, London, 2012.
- B. J. Kubica, *Interval Methods for Solving Nonlinear Constraint Satisfaction, Optimization, and Similar Problems: from Inequalities Systems to Game Solutions*, Springer, Cham, Switzerland, 2019.
- G. Mayer, *Interval Analysis and Automatic Result Verification*, de Gruyter, Berlin, 2017.
- R. E. Moore, R. B. Kearfott, and M. J. Cloud, *Introduction to Interval Analysis*, SIAM, Philadelphia, 2009.

## 6. Acknowledgments

This work was supported in part by:

- National Science Foundation grants 1623190, HRD-1834620, HRD-2034030, and EAR-2225395;
- AT&T Fellowship in Information Technology;
- program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478, and
- a grant from the Hungarian National Research, Development and Innovation Office (NRDI).