

# Why biological neurons are most effective at the border of the set of possible states

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## Formulation of the problem

- Recent research shows that the brain's performance is the best when the neurons' states are the border of the set of possible states.
- How can we explain this phenomenon?

## Our explanation

- To explain this phenomenon, let us recall some basic facts from calculus.
- In general, the maximum of a continuous function  $f(x_1, \dots, x_n)$  on a closed domain  $D$  is attained:
  - either at the interior of this domain
  - or on the domain's border.
- It is reasonable to assume that the optimized function is smooth.
- Let us consider the case when the maximum is attained at the interior point.
- Then, according to calculus, all partial derivatives of this function should be equal to 0:

$$\frac{\partial f}{\partial x_1} = \frac{\partial f}{\partial x_2} = \dots = \frac{\partial f}{\partial x_n} = 0.$$

- In other words, in this case, the point at which the maximum is attained is one of the stationary points of the function  $f(x_1, \dots, x_n)$ .
- A function usually has a small number of stationary points such as local and global minima and maxima.
- When the domain  $D$  is small, the probability that this domain happens to contain one of these stationary points is small.
- So, in most cases, the maximum of a smooth function on a small domain is attained at the border of this domain.
- Let us apply this general idea to our case.
- Neurons – like many other biological systems – are very vulnerable to outside changes.

## Our explanation (cont-d)

- They can only operate within a small range of temperatures, and a small range of other parameters.
- Thus, the set  $D$  of possible states is indeed very small in comparison with the set of all theoretically possible states.
- Most of the physically possible states do not support the living cells.
- So, a natural conclusion is that:
  - the optimal behavior – i.e., the behavior for which the effectiveness  $f(x_1, \dots, x_n)$  attains its largest possible value
  - is indeed attained at the border of this set  $D$ .

## Comment

- This phenomenon is ubiquitous, it is not limited to the brain.
- For example, the optimal regime of a nuclear power station is:
  - when its state is at the edge of instability, i.e.,
  - when the reproduction rate of neutrons is exactly 1.
- A more mundane example – when we walk at a usual speed:
  - we lift a foot up and forward and thus, get an unstable state, on the edge of falling down,
  - and we are saved by putting the foot down.
- The intermediate state is indeed unstable, it is easy to fall down if some obstacle is suddenly encountered when the foot is up.

## Recent references

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## Recent references (cont-d)

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## Older references

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## Acknowledgments

This work was supported in part:

- by the US National Science Foundation grants:
  - 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science),
  - HRD-1834620 and HRD-2034030 (CAHSI Includes),
  - EAR-2225395 (Center for Collective Impact in Earthquake Science C-CIES),
- by the AT&T Fellowship in Information Technology, and
- by the German Research Foundation.