Why biological neurons are most effective at the border of the set of possible states

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1. Formulation of the problem

- Recent research shows that the brain's performance is the best when the neurons' states are the border of the set of possible states.
- How can we explain this phenomenon?

2. Our explanation

- To explain this phenomenon, let us recall some basic facts from calculus.
- In general, the maximum of a continuous function $f(x_1, \ldots, x_n)$ on a closed domain D is attained:
  - either at the interior of this domain
  - or on the domain's border.
- It is reasonable to assume that the optimized function is smooth.
- Let us consider the case when the maximum is attained at the interior point.
- Then, according to calculus, all partial derivatives of this function should be equal to 0:

$$\frac{\partial f}{\partial x_1} = \frac{\partial f}{\partial x_2} = \ldots = \frac{\partial f}{\partial x_n} = 0.$$

3. Our explanation (cont-d)

- In other words, in this case, the point at which the maximum is attained is one of the stationary points of the function $f(x_1, \ldots, x_n)$.
- A function usually has a small number of stationary points such as local and global minima and maxima.
- When the domain D is small, the probability that this domain happens to contain one of these stationary points is small.
- So, in most cases, the maximum of a smooth function on a small domain is attained at the border of this domain.
- Let us apply this general idea to our case.
- Neurons – like many other biological systems – are very vulnerable to outside changes.
- They can only operate within a small range of temperatures, and a small range of other parameters.

4. Our explanation (cont-d)

- Thus, the set D of possible states is indeed very small in comparison with the set of all theoretically possible states.
- Most of the physically possible states do not support the living cells.
- So, a natural conclusion is that:
  - the optimal behavior – i.e., the behavior for which the effectiveness $f(x_1, \ldots, x_n)$ attains its largest possible value
  - is indeed attained at the border of this set D.

5. Comment

- This phenomenon is ubiquitous, it is not limited to the brain.
- For example, the optimal regime of a nuclear power station is:
  - when its state is at the edge of instability, i.e.,
  - when the reproduction rate of neurons is exactly 1.
- A more mundane example – when we walk at a usual speed:
  - we lift a foot up and forward and thus, get an unstable state, on the edge of falling down,
  - and we are saved by putting the foot down.
- The intermediate state is indeed unstable, it is easy to fall down if some obstacle is suddenly encountered when the foot is up.

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