Quantum Ideas in Economics Beyond Quantum Econometrics

Vladik Kreinovich[1], Hung T. Nguyen[2,3], and
Songsak Sriboonchitta[3]

[1]Department of Computer Science, University of Texas at El Paso
El Paso, Texas, USA, USA, vladik@utep.edu
[2]Department of Mathematical Sciences, New Mexico State University
Las Cruces, New Mexico 88002, USA, hunguyne@nmsu.edu
Faculty of Economics, Chiang Mai University
Chiang Mai 50200, Thailand, songsakecon@gmail.com

1. General Problems

- In most practical problems:
  - once we have a candidate for a solution,
  - we can feasibly check whether this candidate is indeed a solution.
- For example, in mathematics, it is often difficult to find a proof of a statement or of its negation; however:
  - once someone produces what intends to be a detailed proof,
  - it is feasible for a referee to check that all the steps in this text are indeed correct and thus,
  - that the text does indeed constitute a proof;
  - we can even use a computer-based system for this checking.

2. Examples of Problems

- Similarly, in physics:
  - it is often difficult to find a formula that described the observed phenomena, but
  - once such a formula is proposed, one can feasibly check whether all observations satisfy it.
- In engineering, it is often difficult to come up with a design that satisfies all the given specifications; but:
  - once a design is produced,
  - we can use software packages to check that this design indeed satisfies the specifications.

3. Class NP

- For example, we can check:
  - that the designed airplane is indeed stable under allowable winds,
  - that the corresponding stresses do not exceed the prescribed level, etc.
- Problems for which we can feasibly check whether a candidate is indeed a solution are known as NP.
- The abbreviation NP stands for *Non-deterministic Polynomial*, where:
  - "non-deterministic" means that we are allowed to guess, and
  - "polynomial" means that once a guess is produced, checking takes polynomial time;
  - such polynomial bounds are a formal description of feasibility.

4. NP and Beyond

- Not all practical problems belong to the class NP.
- For example:
  - if we want to find an *optimal* design,
  - then, in general, it is not easy to check that a given guess is optimal:
  - for that, we would need to compare it with an unfeasible number of all possible designs.
- Similarly, in multi-step conflict situations:
  - it is not easy to check whether a given move is winning or not;
  - checking it would require going over all possible counter-moves of the opposite side.
- However, many practical problem are indeed problems from the class NP.

5. P$\stackrel{?}{=}$NP: An Open Problem

- It is still not known whether we can solve all problems from the class NP is feasible (polynomial) time.
- This is the famous open problem of whether:
  - the class NP is equal to
  - the class P of all the problems that can be solved feasibly (i.e., in polynomial time).
- Most computer scientists believe that NP is different from P.
- The fact that we do not know whether NP is different from P means that:
  - there is no problem from the class NP
  - for which we have proven that this problem cannot be solved in polynomial time.

6. NP-Complete Problems

- There are problems from the class NP which are as hard as possible within this class, in the sense that:
  - every other problem from the class NP
  - can be feasibly reduced to this problem.
- Such problems are known as *NP-complete.*
- Many problems of solving non-linear equations have been proven to be NP-complete.
- The 1st problem for which NP-completeness was proven was *propositional satisfiability (SAT)*:
- Given a *propositional formula* F, i.e., a formula obtained
  - from propositional ("yes"-"no") variables v_i
  - by using propositional connectives & (and), $\vee$ (or), and $\neg$ (not).

7. NP-Complete Problems (cont-d)

- Example: $F = (v_1 \vee v_2 \vee \neg v_3) \,\&\, (\neg v_1 \vee v_2)$.
- Find the values of the variables v_i that make the formula F true.
- Here, a reduction of a problem A to problem B means that:
  - for every instance a of the problem A,
  - we can feasibly compute an appropriate instance b of the problem B.
- Then,
  - once we have a solution to the instance b,
  - we can feasibly transform this solution into a solution to the original instance a.
- Let us give a simple example of reduction.

8. Reduction

- Solving an equation $p \cdot x^4 + q \cdot x + r = 0$ can be reduced to $p \cdot y^2 + q \cdot y + r = 0$.
- If y is a solution to the quadratic equation, then $x = \pm\sqrt{y}$ solves the original equation.
- So, once we know that a problem is NP-complete, then:
  - any good algorithm for solving this problem
  - automatically becomes a good algorithm for solving all other problems from the class NP.
- This is not just a theoretical possibility:
  - efficient tools for solving the propositional satisfiability problem (known as *SAT-solvers*)
  - are now used to solve many problems from different application areas.

9. Why Quantum Ideas in Economics

- From this viewpoint, econometrics has many complex problems.
- Sometimes, we do not have efficient algorithms for solving these problems.
- In this case, due to the above reduction, it is reasonable:
  - to look for other complex (NP-complete) problem, and
  - see if known algorithms for solving these other problems can be applied to economics.
- Where can we find such other problems?
- Most of the practical problems deal with the physical world.

10. Why Quantum Ideas in Economics (cont-d)

- Thus, it is reasonable to look into physics for examples of complex problems with known efficient algorithms.
- It is known that adding quantum effects makes problems more complex.
- Thus, if we look for complex problems in physics, it is reasonable to look for problems of quantum physics.
- So, we arrive at the idea of trying to see if we can
  - apply known algorithms for solving complex problem of quantum physics
  - to solve complex economics-related problems.

11. Quantum Econometrics: What Is Known

- The idea of using quantum techniques to solve economics problems has been successful.
- The corresponding techniques are known as *quantum econometrics*.
- These techniques and their numerous applications are described, e.g., in the seminal book (Baaquie 2004).
- This book emphasizes that quantum econometrics is based on:
  - a *mathematical* similarity of equations,
  - *not* on any similarity between physical ideas of quantum physics and economics ideas.

12. Our Idea and What We Do in This Talk

- Quantum ideas have been very successful in econometric applications.
- This makes us think that there may be deeper reasons for the mathematical similarity.
- In other words, there is indeed some relation between physical ideas of quantum physics and economics.
- In this talk, we show that there is indeed such a relation.

13. Main Ideas Behind Quantum Physics: A Brief Reminder

- The main objective of physics is to learn the state of the physical world and to predict its future state.
- The information about the current state of the physical world comes from measurements.
- To get the most information about the world, we want to make the measurements as accurate as possible.
- This means, in particular:
  - that the measurements should disturb the measured object as little as possible,
  - since each such disturbance changes the state of the object.
- Traditional physics is the physics of *macro-world*, the physics of objects of macro-size.

14. Quantum Physics (cont-d)

- For such objects, it is usually possible to measure them while disturbing them as little as possible.
- For example, we can measure the distance to an object:
  - by sending an ultrasound signal towards the object and
  - by measuring the time it takes for this signal to get to the object, get reflected, and come back.
- We can also perform a similar measurement by sending a laser beam.
- In both cases, we can use relatively weak signals, so that the measured object is not affected by this signal.
- However, as we study smaller and smaller objects, this becomes more and more complicated.

15. Quantum Physics (cont-d)

- When we send a measuring signal to a body consisting of $\approx 10^{23}$ particles:
  - we can have a relatively very weak signal
  - whose effect on the multi-particle body of interest is small.
- However, the situation drastically changes if we consider *micro-objects*.
- To measure the location of an elementary particle:
  - we need to send another particle – e.g., a photon
  - to interact with the particle of interest.
- In this case, the signal that we send is of approximately of the same size as the object itself.
- There is thus no way that we can ignore the effect of this signal on the measured object.

16. Quantum Physics (final)

- In other words, in the micro-world, when we perform a measurement on an object, we change this object.
- This is one of the main features of the micro-world – known as a the quantum world – that:
  - no matter how much we try,
  - we cannot avoid changing the state;
  - whenever we measure the state, we change it.

17. There Is a Similar Idea in Economics

- At first glance, economics is a macro-object:
  - when we measure GDP or unemployment,
  - we do not change it, the value remains very accurate.
- However:
  - econometrics is *not* just measuring different parameters of economics,
  - econometrics is about *discovering new dependencies* that describe the economic data.
- From this viewpoint, econometrics has exactly the same effect as quantum physics:
  - once we discover a new dependence,
  - the situation changes.
- Indeed, let us consider a simplified example.

18. Similar Idea in Economics (cont-d)

- Suppose that a researcher finds out a better way:
  - to predict the price $x(t+2)$ of a certain financial instrument two days from today
  - based on the past prices $x(t)$, $y(t)$, ..., $x(t-1)$, $y(t-1)$, ...
- It is known that the stock values sometimes change drastically: $x(t_0+2) \gg x(t_0), x(t_0+1)$.
- When we did not know the dependence, we could not predict this increase.
- However, since we now know the dependence, the traders know that the price will rise.

19. Similar Idea in Economics (cont-d)

- Therefore, they will start buying this stock:
  - until its price rises, in day $t_0 + 1$, to $\rho \cdot x(t_0 + 2)$,
  - where ρ is a discount that takes into account one day difference.
- This will change the next day's stock price $x(t_0 + 1)$:
  - from the previously predicted value

$$x(t_0 + 1) \ll x(t_0 + 2)$$

  - to a new value $x(t_0 + 1) \approx x(t_0 + 2)$.

20. Similar Idea in Economics (cont-d)

- So:
  - while the model worked perfectly well until it was discovered, once it is discovered,
  - it longer provides correct predictions,
  - because the stock traders take this model into account when trading and
  - thus, change the dynamics of the system and consequently, modify stock prices.
- Similarly, in situations in which the model originally predicted drastic decreases in stock prices,
  - once the model becomes known,
  - it no longer provides accurate predictions.
- This is an exact analog of the quantum physics phenomenon.

21. Similar Idea in Economics (final)

- In quantum physics:
  - once you learn the value of a quantity describing the object,
  - the actual value of this quantity changes, and
  - the known value is no longer a perfect description of the current state of the particle.
- Similarly, in economics:
  - once we discover the previously unknown dependence between economic quantities,
  - this changes the dynamics of trade and
  - thus, the dependence – which worked well in the past – stops working, at least stops being accurate.
- This fundamental similarity may be the reason why quantum techniques were helpful in economics.

22. Comment

- In both cases, the size of the effect depends on the relative size of the object.
- In quantum physics:
  - the effect of measurement on a micro-size body can be minuscule,
  - while for micro-size body, the effect is very drastic.
- Similarly, in economics:
  - if only one person knows the dependence and uses it to buy and sell small amounts of stock,
  - the effect on the stock market will be small.

23. Comment (cont-d)

- However, nowadays,
  - with financial companies actively investing in data analytics,
  - a dependence uncovered by one researcher cannot be kept secret for long.
- It will inevitably (and very soon) be discovered by others as well.
- Once this happens, the effect on the stock market will become large.
- And this will invalidate the original dependence.

24. Acknowledgments

- We acknowledge the partial support of:
  - the Center of Excellence in Econometrics, Faculty of Economics,
  - Chiang Mai University, Thailand.
- This work was also supported in part by the US National Science Foundation grant HRD-1242122.