Relativistic Effects Can Be Used to Achieve a Universal Square-Root (Or Even Faster) Computation Speedup

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1. Need for Fast Computations

- At first glance, the situation with computing speed is very good.
- The number of computational operations per second has grown exponentially fast, and continues to grow.
- Faster and faster high performance computers are being designed and built all the time.
- The only reason why they are not built even faster is the cost limitations.
- However, there are still some challenging practical problems that cannot yet been solved now.
- An example of such a problem is predicting where a tornado will go in the next 15 minutes.
- At present, this tornado prediction problem can be solved in a few hours on a high performance computer.

2. Need for Fast Computations (cont-d)

- However, by then, it will be too late.
- As a result, during the tornado season, broad warning are often so frequent that people often ignore them.
- And they become victims when the tornado hits their homes.
- There are many other problems like this.

3. What Can We Do – In Addition to What Is Being Done

- Computer engineers and computer scientists are well aware of the need for faster computations.
- So computer engineers are working on new hardware that will enable faster computations.
- Computer scientists are developing new faster algorithms for solving different problems.
- Some of the hardware efforts are based:
  - on the same physical and engineering principles
  - on which the current computers operate.
- Some efforts aim to involve different physical phenomena – such as quantum computing.
- Can we use other physical phenomena as well?

4. What Can We Do (cont-d)

- We are talking about speeding up computations, i.e., about time.
- So a natural place to look for such physical phenomena is to look for physical effects that:
  - change the rate of different physical processes,
  - i.e., make them run faster or slower.
- In this paper: we will show how physical phenomena can be used to further speed up computations.

5. Physical Phenomena That Change the Rate of Physical Processes: A Brief Reminder

- Unfortunately for computations, there are no physical processes that *speed up* all physical processes.
- However, there are two physical processes that *slow down* all physical processes.
- First, according to Special Relativity, if we travel with some speed v, then all the processes slow down.
- The time interval s registered by the observer moving with the speed v is called the proper time interval.
- It is related to the time interval t measured by the immobile observer by the formula $s = t \cdot \sqrt{1 - \frac{v^2}{c^2}}$.
- Here c denotes the speed of light.
- The closer the observer's speed v to the speed to the speed of light c, the larger this slow-down.

6. Physical Phenomena (cont-d)

- Second, according to General Relativity Theory, in the gravitational field, time also slows down.
- For immobile observer, the proper time interval s is equal to $s = \sqrt{g_{00}} \cdot t$.
- Here g_{00} is the 00-component of the metric tensor g_{ij} that describes the geometry of space-time.
- In the spherically symmetric (Schwarzschild) solution, we have $g_{00} = 1 - \dfrac{r_s}{r}$, where:
  - r is the distance from the center of the gravitating body and
  - $r_s \stackrel{\text{def}}{=} \dfrac{2G \cdot M}{c^2}$, where G is the gravitational constant and M is the mass of the central body.
- Both slow-down effects have been experimentally confirmed with high accuracy.

7. How We Can Use These Phenomena to Speed up Computations

- If these phenomena would speed up all the processes, then it would be easy to speed up computations:
  - move the computers with a high speed and/or place them in a strong gravitational field,
  - and we would this get computations faster.
- In reality, these phenomena slow down all the processes, not speed them up.
- So, if we place computers in such a slowed-time environment, we will only slow down the computations.

8. How to Speed up Computations (cont-d)

- However, we *can* speed up computations if we do the opposite:
  - keep computers in a relatively immobile place with a reasonably low gravitational field, and
  - place our whole civilization in a fast moving body and/or in a strong gravitational field.
- In this case, in terms of the computers themselves, computations will continue at the same speed, but:
  - since our time will be slowed down,
  - we will observe much more computational steps in the same interval of proper time,
  - i.e., time as measured by our slowed-down civilization.
- In this talk, we analyze what speed up we can obtain in this way.

9. How to Use Special Relativistic Effects for a Speed-Up: Reminder

- To get a speed-up, we can:
  - place the computer at the center, and
  - start moving around this computer at a speed close to the speed of light.
- We cannot immediately reach the speed of light or the desired trajectory radius.
- So, we need to gradually increase our speed and the radius.
- Let $v(t)$ denote our speed at time t, and let $R(t)$ denote the radius of our trajectory at moment t.

10. Analysis of the Problem

- According to Relativity Theory:
  - a change ds in proper time
  - is related to the change dt in coordinate time (as measured by the computer clock) as $ds = dt\cdot\sqrt{1-\dfrac{v^2(t)}{c^2}}$.
- To make civilization with rest energy E_0 move with this speed, we need the energy $E(t) = \dfrac{E_0}{\sqrt{1-\dfrac{v^2(t)}{c^2}}}$.
- Thus, we can say that $ds = dt\cdot\dfrac{E_0}{E(t)}$.
- We need to keep acceleration experienced by all moving persons at the usual Earth level g_0.

11. Analysis of the Problem (cont-d)

- When a body follows a circular orbit with velocity $v(t)$ and radius $R(t)$, it experiences acceleration
$$\frac{d^2x}{dt^2} = \frac{v^2(t)}{R(t)}.$$
- Since the velocity $v(t)$ is close to the speed of light $v(t) \approx c$, we conclude that $\dfrac{d^2x}{dt^2} = \dfrac{c^2}{R(t)}$.
- Substituting $dt = ds \cdot \dfrac{E(t)}{E_0}$ into this formula, we conclude that $\dfrac{E_0^2}{E^2(t)} \cdot \dfrac{d^2x}{ds^2} = \dfrac{c^2}{R(t)}$.
- Here, the experienced acceleration $\dfrac{d^2x}{ds^2}$ should be equal to the usual Earth one g_0: $\dfrac{E_0^2}{E^2(t)} \cdot g_0 = \dfrac{c^2}{R(t)}$.

12. Analysis of the Problem (cont-d)

- Thus, the speed-up is $\dfrac{E(t)}{E_0} = c \cdot \sqrt{\dfrac{R(t)}{g_0}}$.
- The larger $R(t)$, the larger the speed-up.
- All the speeds are limited by the speed of light.
- Thus, we have $R(t) \le v_0 \cdot t$, where $v_0 < c$ is the speed with which we increase the radius.
- To increase the speed-up effect, let us consider the case when $R(t) = v_0 \cdot t$.
- In this case, the speedup has the form $\dfrac{E(t)}{E_0} = C \cdot \sqrt{t}$.
- Here we denoted $C \stackrel{\text{def}}{=} \dfrac{c \cdot \sqrt{v_0}}{\sqrt{g_0}}$.

13. Analysis of the Problem (cont-d)

- Thus, we get $\dfrac{ds}{dt} = \dfrac{E_0}{E(t)} = C^{-1} \cdot t^{-1/2}$, hence

$$ds = C^{-1} \cdot dt \cdot t^{-1/2}.$$

- Integrating both sides, we conclude that $s = 2C^{-1} \cdot \sqrt{t}$.
- Thus, we arrive at the following speed-up scheme.

14. Resulting Speedup Scheme

- To speed up computations, we place computers where they are now, and start moving the whole civilization.
- At any given moment of time t, we move the civilization at a circle of radius $R(t) = v_0 \cdot t$.
- Here, $v_0 < c$ is some pre-determined radial speed.
- The speed $v(t)$ is determined by the formula
$$\frac{E_0^2}{E^2(t)} = 1 - \frac{v^2(t)}{c^2} = \frac{c^2}{R(t) \cdot g_0} = \frac{c^2}{v_0 \cdot g_0 \cdot t}.$$
- Hence $v(t) = c \cdot \sqrt{1 - \dfrac{c^2}{v_0 \cdot g_0 \cdot t}}$.
- The proper time s is related to coordinate time t as $s = 2C^{-1} \cdot \sqrt{t}$, where $C = \dfrac{c \cdot \sqrt{v_0}}{\sqrt{g_0}}$.
- Thus, we indeed get a square-root speedup.

15. This Is All We Can Get

- Note that this square root speedup is all we can gain.
- A further speedup would require having accelerations much higher than our usual level g_0.

16. How Realistic Is This Scheme?

- How big a radius do we need to reach a reasonable speedup?
- As we will show, the corresponding radius is – by astronomical standards – quite reasonable.
- Indeed, for $E(t) \approx E_0$, the above formulas relating $E(t)$ and $R(t)$ leads to

$$R(t) = \frac{c^2}{g_0} \approx \frac{(3 \cdot 10^8 \text{ m/sec})^2}{10 \text{ m/sec}^2} = 9 \cdot 10^{15} \text{ m}.$$

- This radius can be compared with a light year – the distance that the light travels in 1 year – which is:

$$\approx (3 \cdot 10^8 \text{ m/sec}) \cdot (3 \cdot 10^7 \text{ sec/year}) \cdot (1 \text{ year}) = 9 \cdot 10^{15} \text{ m}.$$

- So for $E(t) = E_0$, the radius should be about 1 light year.

17. How Realistic Is This Scheme (cont-d)

- With a speed-up $E(t)/E_0$, the radius grows as the square of this speed-up.
- So, to get an order of magnitude (10 times) speedup, we need an orbit of radius $10^2 = 100$ light years.
- This means reaching to the nearest stars.
- To get a two orders of magnitude (100 times) speedup, we need an orbit of radius $100^2 = 10^4$ light years.
- This almost brings us to the edge of our Galaxy.
- To get a three orders of magnitude (1000 times) speedup, we need an orbit of radius $1000^2 = 10^6$ light years.
- The largest orbit has the radius of the Universe $R(t) \approx$ 20 billion $= 2 \cdot 10^{10}$ light years.
- We can then get $\sqrt{2 \cdot 10^{10}} \approx 1.5 \cdot 10^5$ speedup.

18. This Is Similar to a Quantum Speedup

- This is similar to the speedup of Grover's quantum algorithm for search in an unsorted array.
- The difference is that:
  - in quantum computing, the speedup is limited to search in an unsorted array, while
  - in the above special-relativity scheme, we get the same speedup for *all* possible computations.

19. Comment

- In Russia, to ring the church bells, the monks move the bell's "tongue".
- In Western Europe, they move the bell itself.
- This example is often used in Russian papers on algorithm efficiency, with an emphasis on the fact that,
  - in principle, it is possible to use a third way to ring the bell:
  - by shaking the whole bell tower.
- This third way is mentioned simply as a joke.
- However, as the above computations show, this is exactly what we are proposing here.
- We cannot reach a speedup by making the computer move, so we move the whole civilization.

20. Speculation

- In this scheme, a civilization rotates around a center, increasing its radius as it goes – follows a spiral.
- In this process, to remaining accelerating, the civilization needs to gain more and more kinetic energy $E(t)$.
- The only way to get this energy is to burn all the burnable matter that it encounters along its trajectory.
- As a result, along the trajectory, where the matter has been burned, we have low-density areas.
- Thus, we are left with spiral-shaped low-density areas starting from some central point.
- But this is exactly how our Galaxy – and many other spiral galaxies – look like.
- So maybe this is how spiral galaxies acquired their current shape?

21. Possible General-Relativity Speed-Up: Idea

- We keep the computers were they are now, and place the whole civilization in a strong gravitational field.
- Then our proper time will slow down.
- Thus, the computations that
  - take the same coordinate time t
  - will require, in terms of our proper time s, much fewer seconds.

22. Analysis of the Problem

- According to the Schwarzschild's formula:
  - for the gravitational field of a symmetric body of mass $M(t)$ at a distance $R(t)$ from the center,
  - for an immobile body, we have $ds^2 = g_{00} \cdot dt^2$, where $g_{00}(t) = 1 - \dfrac{2G \cdot M(t)}{c^2 \cdot R(t)}$.
- So, the slow-down $\varepsilon(t) \stackrel{\text{def}}{=} \dfrac{ds}{dt}$ is equal to

$$\varepsilon(t) = \sqrt{g_{00}(t)} = \sqrt{1 - \frac{2G \cdot M(t)}{c^2 \cdot R(t)}}.$$

- We want a good speedup, with $\varepsilon(t) \approx 0$, so we should have $M(t) \approx \dfrac{c^2 \cdot R(t)}{2G}$.

23. Analysis of the Problem (cont-d)

- The coordinate acceleration is equal to $\dfrac{d^2x}{dt^2} = \dfrac{G \cdot M(t)}{R^2(t)}$.
- Substituting the above expression for $M(t)$ into this formula, we conclude that
$$\frac{d^2x}{dt^2} = \frac{c^2 \cdot R(t)}{2R^2(t)} = \frac{c^2}{2R(t)}.$$
- The observed acceleration thus takes the form
$$\frac{d^2x}{ds^2} = \frac{d^2x}{dt^2} \cdot \left(\frac{dt}{ds}\right)^2 = \frac{c^2}{2R(t)} \cdot \frac{1}{\varepsilon^2(t)}.$$
- This acceleration should be equal to the usual Earth's acceleration g_0: $\dfrac{c^2}{2R(t)} \cdot \dfrac{1}{\varepsilon^2(t)} = g_0$.
- Thus $\varepsilon(t) = \dfrac{c}{\sqrt{2R(t) \cdot g_0}}$.

24. Analysis of the Problem (cont-d)

- So, to get faster and faster computations, we need:
  - to constantly increase $R(t)$,
  - and thus, to increase the mass $M(t)$ which is proportional to $R(t)$.
- Similarly to the special relativity case, $R(t)$ cannot grow faster than linearly, so we have $R(t) = v_0 \cdot t$.
- So, the speed-up is proportional to $\varepsilon(t) \sim t^{-1/2}$.
- So, similarly to the special relativity case, we get a square-root speedup.

25. Resulting Speedup Scheme

- To speed up computations:
  - we place computers where they are now, and
  - move at a distance $R(t) = v_0 \cdot t$ from a body of a constantly increasing mass $M(t) = \dfrac{c^2 \cdot R(t)}{2G}$,
  - where G is the gravitational constant.
- We ourselves need to continually increase the corresponding mass.
- In this scheme, we also get a square-root speedup.
- Please note that, similarly to the special relativity scheme, this square root speedup is all we can gain.
- A further speedup would require having accelerations much higher than our usual level g_0.

26. Astrophysical Comment

- There is a threshold of masses after which a body with a sufficiently large mass becomes a black hole.
- Thus, in this scheme, after some time, the civilization is close to a black hole.

27. Ideally, We Should Use Both Speedups

- Moving at a speed close to the speed of light decreases the proper time:
  - from the original value t
  - to a much smaller amount $s \sim \sqrt{t}$.
- Similarly, a location near a black hole also decreases the observable computation time to $s \sim \sqrt{t}$.
- Thus, it makes sense to combine these two schemes – i.e.:
  - place ourselves near an ever-increasing black hole and
  - move (together with this black hole) at a speed close to the speed of light.
- Then, we will replace the perceived computation time from T to $\sqrt{\sqrt{T}} = \sqrt[4]{T}$.

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