

Intuitionistic Fuzzy Techniques, It from Bit, 10- or 11-Dimensional Quantum Field Theory, and Color Optical Computing

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1. The main idea behind quantum physics

- In Newtonian physics, the future state of the world is uniquely determined by its current state:
 - once we know how all the forces depend on the location and velocities of all the objects,
 - we can use Newton's equations to predict the future state of the system.
- For example:
 - once we know the current positions and velocities of all celestial bodies,
 - we can predict their positions hundreds of years ahead.
- And indeed, we can predict Solar eclipses, comet appearances, etc., many decades and even centuries ahead.
- At the end of the 19th century, it turned out the Newtonian predictions were not perfectly accurate.

2. The main idea behind quantum physics (cont-d)

- For example, the actual location of Mercury deviated from the predicted one by 43 angular seconds every 100 years.
- This discrepancy was resolved by Einstein whose General Relativity theory provides perfect predictions.
- From this viewpoint, the behavior of the macro-world is perfectly deterministic.
- However, it turned out that we cannot make deterministic predictions for objects in the microworld.
- One of the first examples was radioactivity, when atoms spontaneously emit some particles.
- Absolutely identical atoms of the same material emit particles at different times – and it is not possible to predict these times.
- We can, at best, predict the probability that an atom will emit during a given time interval.

3. The main idea behind quantum physics (cont-d)

- The same impossibility of exact prediction was observed for all micro-objects such as molecules, atoms, and elementary particles.
- These observation lead to one of the main ideas behind quantum physics:
 - that nothing is pre-determined,
 - that we can make, at best, only probabilistic predictions.

4. Further development of this idea – from primary to secondary quantization

- In the first formalizations of this idea, the physicists took into account that particles randomly deviate from their Newtonian trajectories.
- To compute the underlying Newtonian trajectories:
 - these formalizations used the original Newtonian deterministic formulas,
 - formulas that describe the dependence of force on the location and velocity of all the objects.
- These formalizations provided very accurate description of the micro-world – e.g., the spectrum of light emitted by different materials.
- However, later, more accurate measurements showed that:
 - there is a small discrepancy
 - between the measurement results and the predictions of these formalizations.

5. Further development of this idea – from primary to secondary quantization (cont-d)

- To explain these discrepancy, physicists again applied the main idea of quantum physics:
 - that nothing is pre-determined,
 - that we can make, at best, only probabilistic predictions.
- In the original formalization, particle trajectories were non-deterministic, but the formulas for the forces were still deterministic.
- The physicists concluded that the forces can also randomly deviate from their Newtonian values.
- They called this *secondary quantization*.
- This way, they got a perfect match between theory and experiment.

6. Need for ternary (and further) quantizations

- So far, secondary quantization explains all observed physical phenomena; however:
 - a mathematical analysis of the corresponding equations
 - shows that we may not be able to compute higher-order terms needed for more accurate predictions.
- Some of these terms have physically meaningless infinite values.
- Even when these terms remain finite, the series that should lead to a more and more accurate predictions actually diverge.
- To solve this problem, several physicists – starting with John Wheeler
 - suggested to again use the main quantum idea:
 - that nothing is pre-determined,
 - that we can make, at best, only probabilistic predictions.
- Specifically, in secondary quantization, force is non-deterministic, we can have different force values with different probability.

7. Need for ternary (and further) quantizations (cont-d)

- However, the formulas for describing these probabilities are deterministic.
- A natural idea is thus to allow that the actual probabilities may differ from these values.
- The same argument can be applied to all other physical assumptions:
 - the structure of space-time in which all physically processes take place,
 - the properties of the particles, etc.
- Once we apply this principle to its logical conclusion, we see that there are no deterministic constraints on a physical model.
- All logically consistent physical models are possible – with some probability.
- From this viewpoint, to understand our Universe, we need to consider all possible – i.e., all logically consistent – physical models.

8. Need for ternary (and further) quantizations (cont-d)

- And some physics explanations were indeed provided this way.
- This idea is known as “it from bit”.
- We can potentially describe all the properties of the physical world (“it”) based on logic.
- Physicists use binary “true-false” logic, corresponding to 0-1 binary digits (bits).
- So, they use “bit” as a catchy shorthand for logical foundations.

9. Need for the third option – in addition to “true” and “false”

- Two-valued logic assumes that every statement is either true or false in the corresponding theory.
- However, since the famous Gödel’s result, it is known that:
 - in each well-defined theory (that is sufficiently strong to contain arithmetic),
 - some statements are independent from this theory:
 - neither this statement nor its negation can be derived from the theory.

10. How is this related to intuitionistic fuzzy approach

- This is in line with the main ideas behind intuitionistic fuzzy logic.
- Indeed, in the traditional fuzzy logic:
 - once we know the degree of confidence μ in a statement,
 - the degree of confidence in its negation is estimated as $1 - \mu$, so that the sum of these two degrees will be exactly 1.
- This means, in effect, that we only consider two possibilities: that the statement is true and that the statement is false.
- In reality, often there is a third possibility: that the expert does not know whether the statement is true or false.

11. How is this related to intuitionistic fuzzy approach (cont-d)

- To capture this third option, intuitionistic fuzzy logic uses two degrees:
 - the degree μ_+ to which the expert believes that the statement is true and
 - the degree μ_- to which the expert believes that the statement is false.
- The sum of these two degrees may be smaller than 1.
- The difference $\mu_0 \stackrel{\text{def}}{=} 1 - \mu_+ - \mu_-$ is the degree to which the expert is not sure whether the given statement is true or false.
- In this case, the sum of the *three* degree is equal to 1: $\mu_+ + \mu_- + \mu_0 = 1$.
- *Comment.* In the following, we will argue that:
 - there is an even deeper relationship
 - between the it-from-bit approach and the intuitionistic fuzzy ideas.

12. We need to go further than three values

- In principle, for each formal theory and for each statement, we can try all possible derivations.
- If one of these derivations leads to the given statement, this statement is true.
- If one of these derivations leads to the negation of the given statement, this means that the statement is false.
- However, this argument implicitly assumes that we have a potentially infinite time.
- In reality, according to modern physics, our Universe has a finite lifetime.
- So there may be not enough time to determine whether a statement is true or not.
- One may naively think that this is only important for very long and/or very complex statements.

13. We need to go further than three values (cont-d)

- However, in reality, even for simple propositional formulas, checking whether they are true or not is, in general, NP-hard.
- This means that:
 - unless $P = NP$ (which most computer scientists believe to be impossible)
 - checking the truth of these formulas requires exponential, astronomical time,
 - which or reasonable size formulas exceeds the lifetime of the Universe.
- From this viewpoint:
 - even if we spend the whole lifetime of the Universe,
 - we may not be able to check whether a statement is true or not.
- Thus, in addition to the cases when the is true (T), false (F), or independent (I), we may have several other options.

14. We need to go further than three values (cont-d)

- We may only know that the statement is either true or independent; we will denote this case by TI.
- We may only know that the statement is either false or independent; we will denote this case by FI.
- We may only know that the statement is either true or false, but it cannot be independent; we will denote this case by TF.
- We may not have any information about the statement, so it may be false, it may be true, and it may be independent; we will denote this case by TFI.
- These are not just theoretical possibilities, such situations did happen in the past.
- Let us give a few examples.
- Most mathematical open problems are of the type TFI.

15. We need to go further than three values (cont-d)

- Let us consider the Continuum Hypothesis – that:
 - there is no cardinality strictly between
 - the cardinalities of the set of natural numbers and of the set of real numbers.
- At first, it was proven that this hypothesis is consistent with set theory, i.e., that it is of the type TI.
- It was later shown that it's actually independent.
- Similarly, for the negation of the Continuum Hypothesis, it was originally only proven that:
 - it is either false or independent,
 - i.e., it was of the type FI.

16. We need to go further than three values (cont-d)

- Finally, for some very large integers:
 - we do not know yet whether they are prime,
 - but we know that the corresponding statement cannot be independent:
 - in principle, we can always check whether a given number is prime by checking that it is not divisible by any smaller number;
 - in this case, we have a statement of type TF.

17. How this is related to intuitionistic fuzzy ideas

- To explain this relation, let us recall the usual fuzzy logic.
- Its main idea is that:
 - in addition to statements that the expert knows to be true and the statements that the expert knows to be false,
 - there are also statements about which the expert is not sure.
- In terms of the degree of confidence μ , this means that:
 - in addition to statements for which $\mu = 1$ and statements for which $\mu = 0$,
 - there are also statements for which $0 < \mu < 1$.
- This description can be viewed as the first – qualitative – approximation to the usual fuzzy logic.
- Similarly, we can consider the first – qualitative – approximation to the intuitionistic fuzzy logic.

18. How this is related to intuitionistic fuzzy ideas (cont-d)

- In the intuitionistic fuzzy logic, we have three degrees of confidence μ_+ , μ_- , and μ_0 instead of a single degree μ .
- So, it makes sense to consider, for each of these three degrees μ_i , similar three options: $\mu_i = 1$, $\mu_i = 0$, and $0 < \mu_i < 1$.
- Since the sum of these three degrees is 1, if one of these degrees is equal to 1, then the other two degrees must be equal to 0.
- With this in mind, we end up with the following possible cases:
 - The case when $\mu_+ = 1$ and $\mu_- = \mu_0 = 0$.
 - This case corresponds to T (“true”).
 - The case when $\mu_- = 1$ and $\mu_+ = \mu_0 = 0$.
 - This case corresponds to F (“false”).
 - The case when $\mu_0 = 1$ and $\mu_+ = \mu_- = 0$.
 - This case corresponds to I (“independent”).

19. How this is related to intuitionistic fuzzy ideas (cont-d)

- The case when $\mu_+ > 0$, $\mu_0 > 0$, and $\mu_- = 0$.
- This case corresponds to TI.
- The case when $\mu_- > 0$, $\mu_0 > 0$, and $\mu_+ = 0$.
- This case corresponds to FI.
- The case when $\mu_+ > 0$, $\mu_- > 0$, and $\mu_0 = 0$.
- This case corresponds to TF.
- Finally, we can have the case when $\mu_+ > 0$, $\mu_0 > 0$, and $\mu_- > 0$.
- This case corresponds to TFI.
- One can see that we have exactly the same cases as in the previous subsection.

20. Color interpretation

- We have three truth values T, F, and I, and we have three basic colors: red (R), green (G), and blue (B).
- It therefore makes sense to use different basic colors to illustrate different truth values.
- Other options like TI can then be represented by the combination of the corresponding colors.
- In principle, we can have any mapping between these two triples, but it makes sense to select the following specific mapping:
- The “true” value T gives us the largest amount of information.
- So it makes sense to associate the truth value T with the color that brings the most information.
- Light is a periodic process consisting of cycles.
- In principle, each cycle can carry a number.

21. Color interpretation (cont-d)

- The more cycles per second, the more numbers – and thus, the more information – we can carry.
- The number of cycles per second is nothing else but the frequency.
- Thus, it makes sense to associate T with the basic color that has the largest frequency – i.e., with color blue (B).
- From I value, we can go to T and we can go to F.
- From this viewpoint, I is close to both T and F, while T and F are not close to each other.
- Thus, it makes sense:
 - to assign the two remaining colors – R and G.
 - so that the color associated with I be closer to the color B associated with T than the color associated with F.

22. Color interpretation (cont-d)

- Among the two remaining colors, G is closer to B than R.
- So it makes sense to associate I with G and F with R.

23. Comment

- The association of “false” (F) with red (R) is also in good accordance with the cultural references.
- In traffic lights, red means prohibited – i.e., false.
- In accounting, red means losing money – which is, of course, negative (= false).
- In pedagogy, red color is usually used to mark mistakes – e.g., false statements.

24. Color optical computing

- Colors not only provide a visual interpretation of truth values.
- They also enable us to perform computations really fast.
- There are several reasons why color optical computing is fast.
- The first two reasons refer to optical computing in general.
- First, according to relativity theory, the speed of all motions – and, in particular, the speed of all communications – is limited by the speed of light.
- Thus, light is the fastest possible process;
- Second, light consists of many particles (photons).
- So there is a huge possibility to parallelize computing by assigning different computations to different parts of the light beam.
- And parallelization is a well-known way to speed up computations.

25. Color optical computing (cont-d)

- Color computing adds an additional possibility of parallelization – and thus, of a speedup.
- Namely, we can simultaneously perform three different computations in lights of three different colors.

26. Need for a quantitative use of intuitionistic fuzzy approach

- Let us illustrate this need on a simple example of, e.g., the case TI.
- In some cases:
 - we have a lot of arguments in favor of T,
 - but we do not yet have a proof of the given statement.
- So, we assign this statement to the class TI.
- In other cases, we may have more arguments that this statement is probably independent.
- This can happen because many similar statements have been proven to be independent.
- Both these situations are of the same type TI, but they are clearly different.
- It is desirable to explicitly describe this difference – e.g., by assigning to each of the two options T and I different degrees of confidence.

27. How can we make the use quantitative

- The desired numerical degrees of confidence are used in intuitionistic fuzzy logic.
- There – similarly to the usual fuzzy logic – each degrees of confidence can take any real value from the interval $[0, 1]$.
- Of course, using all possible real numbers from this interval is an idealization.
- No one can meaningfully distinguish, e.g., between degree 0.80 and degree 0.82.
- We can only meaningfully distinguish between a small finite number of different degrees of confidence.
- So, to get the quantitative description more adequate, it makes sense to select some small positive integer $n > 1$ and consider degrees

$$0, 1/n, 2/n, \dots, (n-1)/n, \text{ and } 1.$$

28. What is the simplest quantitative use

- The larger n , the more complex the corresponding approach.
- We want to get the simplest possible interpretation.
- So, it makes sense to consider the smallest number n that allows us to capture all above-listed qualitative cases,
- Let us analyze what is this smallest n .
- The smallest integer $n > 1$ is $n = 2$.
- In this case, we can only use degrees 0, $1/2$, and 1.
- In this case, we cannot represent the case of TFI, where all three degrees are positive.
- Indeed, in the $n = 2$ case, the smallest sum of positive degrees is $3/2$ – which is larger than 1.
- Since $n = 2$ does not work, let us consider the next integer $n = 3$.
- For this n , it is already possible to cover all the qualitative cases.

29. What is the simplest quantitative use (cont-d)

- Specifically, we have the following options:
 - three cases T, F, and I, for which one of the degrees is 1; in these cases, $\mu = (\mu_+, \mu_-, \mu_0)$ is equal to:

$$(1, 0, 0), \quad (0, 1, 0), \quad (0, 0, 1);$$

- for each of the types TI, FI, and TF, two possible combinations of degrees; in these cases, the triple $\mu = (\mu_+, \mu_-, \mu_0)$ is equal to:

$$\left(\frac{2}{3}, 0, \frac{1}{3}\right), \left(\frac{1}{3}, 0, \frac{2}{3}\right), \left(0, \frac{2}{3}, \frac{1}{3}\right), \left(0, \frac{1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{1}{3}, 0\right), \left(\frac{1}{3}, \frac{2}{3}, 0\right);$$

- for the type TFI, there is one possible triple:

$$\mu = (\mu_+, \mu_-, \mu_0) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

- Overall, there are 10 possible options.

30. The may explain the appearance of 10- or 11-dimensional space-time in quantum field theory

- It is known that:
 - in our usual 4-dimensional space-time – consisting of 3-dimensional space and 1-dimensional time,
 - it is not possible to have a consistent quantum field theory.
- The smallest space-time dimension for which a consistent quantum field theory is possible is 10 or 11.
- This corresponds to 9- or 10-dimensional proper space and 1-dimensional time.
- This conclusion is based on complex mathematical analysis whose physical meaning is not very clear.
- It is therefore desirable to come up with a physically meaningful explanation for this dimension.

31. The above 10-options results leads to a possible explanation

- To provide such an explanation, let us recall some simple facts from quantum physics.
- Specifically, let us describe a quantum analogue of the classical (non-quantum) system that can be in N possible states $s_1, \dots, s_N$.
- In quantum physics, in addition to all these states, the system can also be in a *superposition* state $c_1 \cdot |s_1\rangle + \dots + c_N \cdot |s_N\rangle$.
- Here $c_1, \dots, c_N$ are complex numbers with the only constraint

$$|c_1|^2 + \dots + |c_N|^2 = 1.$$

- This fact is well known and well-used in *quantum computing*.
- There, the quantum analogue of a bit – a system with two possible states 0 and 1 – is a *qubit*, with possible states $c_0 \cdot |0\rangle + c_1 \cdot |1\rangle$.
- Here c_i are complex numbers for which $|c_0|^2 + |c_1|^2 = 1$.

32. The above 10-options results leads to a possible explanation (cont-d)

- In this case, instead of two possible states, we have a 2-dimensional set of states.
- To be more precise, the set of all possible states is a unit sphere in the 2-dimensional complex space.
- In general, a quantum analogue of an system with N possible states in a system with N -dimensional space.
- The above system that has at least 10 possible states – it can have more if consider $n > 3$.
- So, its quantum analogue is a 10-dimensional space.
- Thus, to describe a state of this system at each moment of time, we need a point in a 10-dimensional space.
- This is exactly the same dimension as coming from the complex mathematics of quantum field theory.

33. Comment

- There are other physical explanation why the dimension of proper space in consistent quantum field theories is 9 or 10.
- Let us mention two other explanations.
- One of the reasons why quantum field theory is inconsistent in 3-dimensional space is because:
 - according to relativity theory,
 - elementary particles are point-wise objects.
- As a result:
 - when we compute the overall energy of the electric field generated by a charged particle,
 - we get a physically meaningless infinite value.

34. Comment (cont-d)

- A natural way out is to take into account that:
 - due to the quantum fluctuations,
 - particles are not exactly point-wise,
 - they follow some probability distribution of possible locations.
- Each particle is affected by many possible quantum fluctuations.
- It is known that, in general:
 - the joint effect of a large number of small random disturbances
 - is distributed according to the Gaussian (normal) distribution.
- The corresponding mathematical result – known as the *Central Limit Theorem* – explains the ubiquity of normal distributions.
- A normal distribution in an d -dimensional space is uniquely determined by its mean $m = (m_1, \dots, m_d)$ and its covariance matrix c_{ij} .

35. Comment (cont-d)

- To describe the components of the mean vector and of the covariance matrix, we need

$$d + \frac{d \cdot (d + 1)}{2} \text{ parameters.}$$

- In particular, in the 3-dimensional space, we need $3 + 6 = 9$ parameters.
- Another explanation is related to another well-known problem.
- It is known that space-time is curved.
- However, it is difficult to take this fact into account in a quantum description.
- One way to take this into account is to use another known fact.

36. Comment (cont-d)

- Namely, the equations of General Relativity Theory – that describes our curved space-time:
 - can be derived in the flat (not curved) space-time
 - if we simply take into account that the source of the gravitational field is energy – including the energy of the gravitation field itself.
- From this viewpoint, it makes sense:
 - to embed the curved space-time into a flat one with higher dimension, and
 - to use the usual quantum field theory in this high-dimensional space-time.
- Interestingly:
 - the smallest dimension of a flat space-time into which we can embed (at least locally) any curved 4-dimensional space-time
 - is exactly 10.

37. Comment (cont-d)

- This provides one more explanation of the use of 10-dimensional space.
- The same dimension appears if we consider global embeddings.

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