

How to Reconcile Physical Theories with the Idea of Free Will: From Analysis of a Simple Model to Interval and Fuzzy Approaches

Julio C. Urenda¹ and Olga Kosheleva²

Departments of ¹Mathematical Sciences and ²Teacher Education

University of Texas, El Paso, TX 79968, USA

emails jcurenda@miners.utep.edu, olgak@utep.edu

[The problem of free ...](#)

[Interval and fuzzy ...](#)

[What we plan to describe](#)

[Symmetry](#)

[Symmetries and ...](#)

[Case of a single particle](#)

[Case of two particles](#)

[Case of 3 particles: ...](#)

[General case: analysis](#)

[General conclusion](#)

[Discussion](#)

[Acknowledgments](#)

[This Page](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Page 1 of 15](#)

[Go Back](#)

[Full Screen](#)

[Close](#)

1. Introduction

- *Free will: a natural idea.* If we walk to a corner, then we can turn right or cross the street.
- *Commonsense belief:* it is not possible to predict beforehand what exactly a person will do.
- *In classical physics:*
 - once we know the positions and velocities of all the particles,
 - we can uniquely predict the exact future locations and velocities of all the particles.
- *Problem:* can we reconcile physics with free will?
- *Clarification:* with 10^{23} particles, predictions are not practically possible.
- From the commonsense viewpoint, even a theoretical prediction probability is very disturbing.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 2 of 15

Go Back

Full Screen

Close

2. Is quantum physics an answer?

- At first glance, it may look as if this problem disappears in quantum physics.
- Due to Heisenberg's principle, we cannot exactly predict both the location and the velocity.
- Schroedinger's equations describe how the state ("wave function") $\psi(x, t)$ changes with time t .
- These equations are deterministic
 - once we know the original state $\psi(x, t_0)$,
 - we can uniquely determine the future state.
- So, we can uniquely predict the probabilities.
- In particular, we can predict (at least theoretically) the probability that a person turns right.
- This also contradicts to common sense.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 3 of 15

Go Back

Full Screen

Close

3. The problem of free will in physics has been actively studied in philosophy of physics

- *Mainstream approach*:
 - keep the physics as is;
 - commonsense intuition is faulty.
- *Argument*: quantum mechanics showed that commonsense intuitions are only approximately correct.
- *Alternative approach* (Penrose et al.): we need to modify our physical theories.
- *Problem*: no well-developed physical theory is fully consistent with our free will intuition.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 4 of 15

Go Back

Full Screen

Close

4. Interval and fuzzy approaches: towards reconciliation between physics and free will

- *Traditional approach*: differential equations.
- *Idea*: the rate of change is uniquely determined by the state: $\frac{d\vec{v}_i}{dt} = \vec{F}_i(\vec{r}_1, \dots, \vec{r}_n, \vec{v}_1, \dots, \vec{v}_n)$.
- *Conclusion*: no free will.
- *Corollary*: to get free will, we must allow several possible values of rate of change.
- *Natural idea*: interval of possible values:
$$\frac{dv_{ia}}{dt} \in [\underline{F}_{ia}(\vec{r}_1, \dots, \vec{r}_n, \vec{v}_1, \dots), \overline{F}_{ia}(\vec{r}_1, \dots, \vec{r}_n, \vec{v}_1, \dots)].$$
- *Alternative idea*: several intervals corresponding to different degrees of certainty.
- Such nested intervals can be viewed as α -cuts of a fuzzy set, so we get fuzzy differential inclusions.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 5 of 15

Go Back

Full Screen

Close

5. What we plan to describe

- Our objective is
 - to *reasonably* modify the equations of physics
 - so that it will be possible to make the motion no longer uniquely predictable.
- *In plain terms*: a physically explicit free will would mean that
 - by simply exercising our will,
 - we can actually change the motion of the physical particles.
- We would like to check if this is indeed possible within a meaningful physical theory.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 6 of 15

Go Back

Full Screen

Close

6. Symmetry

- We need a theory which is consistent with free will.
- We want this theory to be physically meaningful.
- In modern physics, one of the most important notions is the notion of *symmetry*.
- The behavior of the physical particles must not change if we simply
 - shift them to a different spatial location,
 - or rotate the whole configuration,
 - or start the experiment at a later time moment.
- Thus, a meaningful physical theory must be invariant w.r.t natural symmetries.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 7 of 15

Go Back

Full Screen

Close

7. Symmetries and conservation laws

- It is known that in physical equations, invariance with respect to symmetries lead to conservation laws:
 - invariance w.r.t. shifts in time means that energy $E = \sum_{i=1}^n \frac{1}{2} \cdot m_i \cdot (\vec{v}_i)^2$ must be preserved;
 - invariance w.r.t spatial shifts means that the (linear) momentum $\vec{p} = \sum_{i=1}^n m_i \cdot \vec{v}_i$ must be preserved;
 - invariance w.r.t. rotations means that the angular momentum $\vec{M} = \sum_{i=1}^n m_i \cdot (\vec{v}_i \times \vec{r}_i)$ must be preserved.
- Thus, we require that these three quantities are preserved in our physical theory.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 8 of 15

Go Back

Full Screen

Close

8. Case of a single particle

- *Situation*: let us start our analysis with the case of a single particle.
- *Fact*: for this particle, the momentum $\vec{p} = m_1 \cdot \vec{v}_1$ is preserved.
- *Conclusion*: the velocity \vec{v}_1 is also preserved.
- *Conclusion*: no matter how much we exercise our will, this particle will not be diverted from its inertial path.
- So, for a single particle, no “true free-will” theory is possible.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 9 of 15

Go Back

Full Screen

Close

9. Case of two particles

- Select the $t = t_0$ center of mass as the coordinates origin: $\frac{m_1 \cdot \vec{r}_1 + m_2 \cdot \vec{r}_2}{m_1 + m_2} = \vec{0}$, hence $\vec{r}_2 = -\frac{m_1}{m_2} \cdot \vec{r}_1$.
- Take a system that (originally) moves with the center: $m_1 \cdot \vec{v}_1 + m_2 \cdot \vec{v}_2 = \vec{0}$.
- Since the momentum $\vec{p} = m_1 \cdot \vec{v}_1 + m_2 \cdot \vec{v}_2$ is preserved, we have $m_1 \cdot \vec{a}_1 + m_2 \cdot \vec{a}_2 = 0$ hence $\vec{a}_2 = -\frac{m_1}{m_2} \cdot \vec{a}_1$.
- Since the angular momentum is preserved, we get $m_1 \cdot (\vec{a}_1 \times \vec{r}_1) + m_2 \cdot (\vec{a}_2 \times \vec{r}_2) = 0$, hence $\vec{a}_1 \times \vec{r}_1 = \vec{0}$.
- Thus, \vec{a}_1 is collinear with \vec{r}_1 .
- Since energy $\sum \frac{\vec{v}_i^2}{2}$ is preserved, we get $\vec{a}_1 \cdot \vec{v}_1 = 0$ hence $\vec{r}_1 \cdot \vec{v}_1 = 0$ – which is in general not true.
- *Conclusion:* no free will for 2-particle systems.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 10 of 15

Go Back

Full Screen

Close

10. Case of 3 particles: analysis

- For 3 particles, invariance leads to linear equations:

$$m_1 \cdot \vec{a}_1 + m_2 \cdot \vec{a}_2 + m_3 \cdot \vec{a}_3 = 0;$$

$$m_1 \cdot (\vec{a}_1 \times \vec{r}_1) + m_2 \cdot (\vec{a}_2 \times \vec{r}_2) + m_3 \cdot (\vec{a}_3 \times \vec{r}_3) = 0;$$

$$m_1 \cdot (\vec{a}_1 \cdot \vec{v}_1) + m_2 \cdot (\vec{a}_2 \cdot \vec{v}_2) + m_3 \cdot (\vec{a}_3 \cdot \vec{v}_3) = 0.$$

- We need to find three 3-D vectors \vec{a}_1 , \vec{a}_2 , and \vec{a}_3 , i.e., $3 \cdot 3 = 9$ scalar unknowns.
- The first two equations are vector equations, each of which has 3 scalar components.
- So overall, we have 7 scalar equations to determine 9 (scalar) unknowns.
- Clearly, a linear system of 7 equations with 9 unknowns has many solutions.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 11 of 15

Go Back

Full Screen

Close

11. General case: analysis

- For p particles, invariance leads to linear equations:

$$m_1 \cdot \vec{a}_1 + \dots + m_p \cdot \vec{a}_p = 0;$$

$$m_1 \cdot (\vec{a}_1 \times \vec{r}_1) + \dots + m_p \cdot (\vec{a}_p \times \vec{r}_p) = 0;$$

$$m_1 \cdot (\vec{a}_1 \cdot \vec{v}_1) + \dots + m_p \cdot (\vec{a}_p \cdot \vec{v}_p) = 0.$$

- We need $3 \cdot p$ scalar parameters to determine p accelerations $\vec{a}_1, \dots, \vec{a}_p$.
- We have the same number of 7 equations to satisfy.
- Since $3 \cdot p \geq 3 \cdot 3 > 7$, the corresponding linear system of equations always has a non-zero solution \vec{a}_i .
- *Conclusion:* for ≥ 3 particles, a “true free-will” physical theory is, in principle, possible.
- *Comment:* for 2 particles, we need to determine 6 scalar unknowns from $7 > 6$ equations.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 12 of 15

Go Back

Full Screen

Close

12. General conclusion

- Physical theories (gravity, electrodynamics, etc.) are based on pairwise interactions between particles.
- *Example:* Newton's gravitation theory,

$$\vec{F}_i = \sum_{j \neq i} \frac{G \cdot m_i \cdot m_j \cdot (\vec{r}_j - \vec{r}_i)}{|\vec{r}_j - \vec{r}_i|^3}.$$

- In such theories, interaction between ≥ 3 bodies reduces to pairwise interaction.
- *Example:* Earth, Sun, and Moon resulting in tides.
- In a free-will theory, we must have triple interactions:

$$\vec{F}_i = \sum_{j \neq i} \vec{F}_{ij}(\vec{r}_i, \vec{r}_j, \vec{v}_i, \vec{v}_j) + \sum_{j \neq i} \sum_{k \neq i} \vec{F}_{ijk}(\vec{r}_i, \vec{r}_j, \vec{r}_k, \vec{v}_i, \vec{v}_j, \vec{v}_k).$$

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 13 of 15

Go Back

Full Screen

Close

13. Discussion

- *Reminder*: for a true free-will theory, we need at least triple interactions.
- Similar cases when triple interactions bring complexity:
 - in decision making, combining 2 opinions is easy, but combining 3 leads to Arrow's paradox;
 - in celestial mechanics, 2-body problem is explicitly solved while a 3-body problem is complex.
- Possible use in fuzzy logic:
 - we normally use binary logical operations, fuzzy analogues of “or”, “and”, etc.;
 - more complex logical operations (e.g., ternary one) are usually reduced to the binary ones;
 - non-reducible ternary operations may lead to a more adequate representation of expert uncertainty.

The problem of free ...

Interval and fuzzy ...

What we plan to describe

Symmetry

Symmetries and ...

Case of a single particle

Case of two particles

Case of 3 particles: ...

General case: analysis

General conclusion

Discussion

Acknowledgments

Title Page



Page 14 of 15

Go Back

Full Screen

Close

14. Acknowledgments

This work was supported in part:

- by NSF grants HRD-0734825, EAR-0225670, and EIA-0080940,
- by Texas Department of Transportation contract No. 0-5453,
- by the Japan Advanced Institute of Science and Technology (JAIST) International Joint Research Grant 2006-08, and
- by the Max Planck Institut für Mathematik.

The authors are thankful

- to Professor John Symons from Philosophy Department and
- to the anonymous referees

for valuable suggestions.

Introduction
Is quantum physics an . . .
The problem of free . . .
Interval and fuzzy . . .
What we plan to describe
Symmetry
Symmetries and . . .
Case of a single particle
Case of two particles
Case of 3 particles: . . .
General case: analysis
General conclusion
Discussion

Acknowledgments

Title Page



Page 15 of 15

Go Back

Full Screen

Close

Quit