On Uncertainty Propagation: A Feasible Algorithm for Checking Whether an Expression Is Equivalent to a Single-Use One (SUE)

Joe Lorkowski, Olga Kosheleva, and Vladik Kreinovich University of Texas at El Paso, El Paso, TX 79968 lorkowski@computer.org, olgak@utep.edu,vladik@utep.edu

Data processed by computers mostly comes from measurements, and measuring real-number values is never exact. Often, the only information that we have about measurement errors is the upper bound d. In this case, based on the measurement result X, the only information that we have about the actual value  $\boldsymbol{x}$  of the measured quantity is that x belongs to the interval [X - d, X + d]. For each data processing algorithms y = f(x1, ..., xn), different values of xi from the corresponding intervals lead, in general, to different result y. To understand the uncertainty of the result of data processing, it is therefore important to find the range of all possible values y. In general, computing such a range is NP-hard, but there is a case when this range can be feasibly computed: the case of SUE, when each variable occurs only once. In this case, if we represent the computation of f as a sequence of elementary arithmetic operations and replace each operation by the corresponding interval operation, we get the exact range. In some cases, the original expression is not SUE, but it can be transformed into the SUE form: e.g., a/(a + b) can be transformed into 1/(1 + b/a). In this talk, we describe a feasible algorithm that checks whether a given expression can be transformed into the SUE form, and if yes, produced the corresponding form.