How Do Degrees of Confidence Change with Time?

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Formulation of the problem. Situations change. As a result, our degrees of confidence in statements based on past experience decrease with time. How can we describe this decrease? If our original degree of confidence was a, what will be our degree of confidence $d_t(a)$ after time t? (It is clear that $d_t(a)$ should be increasing in a and decreasing in t, but there are many such functions.)

Our idea. Let $f_\&(a,b)$ be an "and"-operation, i.e., a function that transforms degrees of confidence a and b in statements A and B into an estimate for our degree of confidence in $A\,\&\,B$. There are two ways to estimate our degree of confidence in $A\,\&\,B$ after time t: we can apply the function d_t to both a and b, and then combine them into $f_\&(d_t(a),d_t(b))$, or we can apply d_t directly to $f_\&(a,b)$, resulting in $d_t(f_\&(a,b))$. It is reasonable to require that these two expressions coincide: $f_\&(d_t(a),d_t(b))=d_t(f_\&(a,b))$.

Simplest case. If $f_\&(a,b)=a\cdot b$, then the above equality becomes $d_t(a\cdot b)=d_t(a)\cdot d_t(b)$. It is known that all monotonic solutions to this equation have the form $d_t(a)=a^{p(t)}$ for some $p(t)$.

The dependence on t can be found if we take into account that the decrease during time $t=t_1+t_2$ can also be computed in two ways: directly, as $a^{p(t_1+t_2)}$, or by first considering decrease during t_1 ($a\to a'=a^{p(t_1)}$), and then a decrease during time t_2: $a'\to (a')^{p(t_2)}=\left(a^{p(t_1)}\right)^{p(t_2)}=a^{p(t_1)\cdot p(t_2)}$. It is reasonable to require that these two expressions coincide, i.e., that $p(t_1+t_2)=p(t_1)\cdot p(t_2)$. The only monotonic solutions to this equation are $p(t)=\exp(\alpha\cdot t)$, so we get $d_t(p)=a^{\exp(\alpha\cdot t)}$.

Comment. It is worth mentioning that for small t, we get $d_t(p)\approx a+\alpha\cdot t\cdot a\cdot\ln(a)$ and is thus related to entropy $-\sum a_i\cdot\ln(a_i)$.

General case. It is known that every "and"-operation can be approximated, with any accuracy, by a Archimedean one, i.e., by an operation of the type $f_\&(a.b)=g^{-1}(g(a)\cdot g(b))$. Thus, for re-scaled values $a'=g(a)$, we have $f_\&(a',b')=a'\cdot b'$, hence $d_t(a')=(a')^{\exp(\alpha\cdot t)}$ and, in the original scale,

$$d_t(a)=g^{-1}\left((g(a))^{\exp(\alpha\cdot t)}\right).$$