What If We Only Know Hurwicz's Optimism-Pessimism Parameter with Interval Uncertainty?

Jeffrey Hope, Olga Kosheleva, and Vladik Kreinovich
University of Texas at El Paso, El Paso, TX 79968, USA
jshope2@miners.utep.edu, olgak@utep.edu, vladik@utep.edu

Formulation of the problem. In many practical situations, we do not know the exact consequences of each possible action. As a result, instead of single utility value u, we can characterize each possible action by the interval $[\underline{u}, \overline{u}]$ of possible utility values. In such situations, decision theory recommends to select an alternative for which the following combination is the largest: $\alpha \cdot \overline{u} + (1 - \alpha) \cdot \underline{u} = \underline{u} + \alpha \cdot (\overline{u} - \underline{u})$, where the parameter α – known as Hurwicz's *optimism-pessimism* parameter – may be different from different people. The value $\alpha = 1$ corresponds to absolute optimists, the value $\alpha = 0$ describes a complete pessimist, and values between 0 and 1 describe reasonable decision makers.

The parameter α needs to be determined based on a person's preferences and decisions. Often, however, in different situations, the decisions of the same person correspond to somewhat different values α. As a result, instead of a single value α, we have the whole range $[\underline{\alpha}, \overline{\alpha}]$ of possible values. In this case, how should we make decisions?

Our solution. For each $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, the interval $[\underline{u}, \overline{u}]$ is equivalent to $\underline{u} + \alpha \cdot (\overline{u} - \underline{u})$. This expression is monotonic in α, so in general, the original interval $[\underline{u}, \overline{u}]$ is equivalent to the following range of possible values $[\underline{u}_1, \overline{u}_1] = [\underline{u} + \underline{\alpha} \cdot (\overline{u} - \underline{u}), \underline{u} + \overline{\alpha} \cdot (\overline{u} - \underline{u})]$. Similarly, the new interval is equivalent to $[\underline{u}_2, \overline{u}_2] = [\underline{u}_1 + \underline{\alpha} \cdot (\overline{u}_1 - \underline{u}_1), \underline{u}_1 + \overline{\alpha} \cdot (\overline{u}_1 - \underline{u}_1)]$, etc. At each step, the width of the original intervals decreases by the factor $\overline{\alpha} - \underline{\alpha}$: $\overline{u}_k - \underline{u}_k = (\overline{\alpha} - \underline{\alpha})^k \cdot (\overline{u} - \underline{u})$. In the limit, the intervals $[\underline{u}_k, \overline{u}_k]$ tend to a single point

$$u = \underline{u} + \underline{\alpha} \cdot (\overline{u} - \underline{u}) + \underline{\alpha} \cdot (\overline{u} - \underline{u}) \cdot (\overline{\alpha} - \underline{\alpha}) + \underline{\alpha} \cdot (\overline{u} - \underline{u}) \cdot (\overline{\alpha} - \underline{\alpha})^2 + \ldots$$

The corresponding geometric progression adds to $\underline{u} + \alpha \cdot (\overline{u} - \underline{u})$ for $\alpha = \dfrac{\underline{\alpha}}{1 - (\overline{\alpha} - \underline{\alpha})}$. This is the desired equivalent value of α for the case when we know α with interval uncertainty – and this is how we should make decisions in this case.