# Avoiding Einstein-Podolsky-Rosen (EPR) Paradox: Towards a More Physically Adequate Description of a Quantum State

Joseph Bernal[1], Olga Kosheleva[2], and Vladik Kreinovich[2]
[1]Department of Philosophy
El Paso Community College, El Paso, TX 79902, USA
[2]University of Texas at El Paso, El Paso, TX 79968, USA
jberna36@epcc.edu, olgak@utep.edu, vladik@utep.edu

**EPR paradox: brief reminder.** Traditionally, a state of a quantum system is described by a complex-valued *wave function.* For two independent particles in states $\psi_i(x_i)$, the joint state is a product $\psi_1(x_1)\cdot\psi_2(x_2)$. When particles are not independent (= *entangled*), we can have a more complex joint state, such as the state $\dfrac{1}{\sqrt{2}}\cdot(|0_1 0_2\rangle + |1_1 1_2\rangle)$ used in quantum computing.

When we measure the state of Particle 1, then, according to quantum physics, the joint state changed into either into $|0_1 0_2\rangle$ or into $|1_1 1_2\rangle$. Thus, the state of the second particle – as described by the wave function – immediately changed too. When the particles are separated, this action-at-a-distance seems to contradict special relativity, according to which all speeds are limited by the speed of light $c$; this is the essence of the EPR paradox.

**Bohr's explanation.** Niels Bohr explained that while the wave function indeed changes immediately, this process cannot be used for faster-than-light communication: the results of measurements performed on Particle 2 does not change when we perform the measurements on Particle 1.

**Towards a more adequate description of a quantum state.** The EPR paradox shows that wave function, while convenient for computations and predictions, is not always the most physically adequate description of a quantum state. To get a more adequate description, we can take a spatial region $U$ and consider only measurements that depend on what is inside $U$. For example, for a single particle, these measurements only depend on the values $\psi(x)$ for $x\in U$. One can show that in general, to describe the probabilities of different measurement results, it is sufficient to describe the corresponding density matrix $\rho(U)$.

When we limit ourselves to a subregion $U'\subset U$, then $\rho(U')$ is equal to a naturally defined projection of $\rho(U)$: $\rho(U')=\pi_{U\to U'}(\rho(U))$. In mathematical terms, matrices $\rho(U)$ form a *sheaf* – which is thus a more physically adequate representation of the quantum state than the wave function.