Safety Factors in Soil and Pavement Engineering: Theoretical Explanation of Empirical Data

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What is a safety factor. Models are approximations to reality. To describe a complex real-life process by a feasible model, we find the most important factors affecting the process and model them. Thus, we ignore small factors; they may be smaller than the factors that we take into account but they still need to be taken into account if we want to provide guaranteed bounds for the desired quantities. To take these small factors into account, engineers multiply the results of the model by a constant known as the *safety factor*.

Safety factors in soil and pavement engineering: empirical data. In many applications, a safety factor is 2 or smaller. However, in soil and pavement engineering, comparison of the resilient modulus predicted by the corresponding model and the modulus measured by Light Weight Deflectometer shows that, to provide guaranteed bounds, we need a safety factor of 4; see, e.g., [1]. How can we explain this?

Explaining the safety factor of 2: reminder. Let Δ be the model's estimate. When designing the model, we did not take into account some factors. Let's denote the effect of the largest of these factors by Δ_1. The factors that we ignored are smaller than the one we took into account, so $\Delta_1 < \Delta$, i.e., $\Delta_1 \in [0, \Delta]$. We do not have any reason to assume that any value from the interval $[0, \Delta]$ is more frequent than others; thus, it makes sense to assume that Δ_1 is uniformly distributed on $[0, \Delta]$. Then, the average value of Δ_1 is $\Delta/2$.

The next smallest factor Δ_2 is smaller than Δ_1. The same arguments shows that its average value is $\Delta_1/2$, i.e., $\Delta_2 = 2^{-2} \cdot \Delta$. Similarly, $\Delta_k = 2^{-k} \cdot \Delta$, hence the overall estimate is $\Delta + \Delta_1 + \ldots = \Delta + 2^{-1} \cdot \Delta + \ldots + 2^{-k} \cdot \Delta + \ldots = 2\Delta$.

A similar explanation for the safety factor of 4. Empirical data shows that for soil and pavement engineering, 2 is not enough. This means that Δ_1 should be larger than our estimate $\Delta/2$: $\Delta_1 \in [\Delta/2, \Delta]$. In this case, the average value from this interval is $\Delta_1 = (3/4) \cdot \Delta$. Similarly, we get $\Delta_2 = (3/4)^2 \cdot \Delta$, $\Delta_k = (3/4)^k \cdot \Delta$ and thus,

$$\Delta + \Delta_1 + \ldots + \Delta_k + \ldots = \Delta \cdot (1 + 3/4 + \ldots + (3/4)^k + \ldots) = \Delta/(1 - 3/4) = 4\Delta.$$

[1] M. Mazari, E. Navarro, I. Abdallah, and S. Nazarian, "Comparison of numerical and experimental responses of pavement systems using various resilient modulus models", *Soils and Foundations*, 2014, Vol. 54, No. 1, pp. 36–44.