Experimental Determination of Mechanical Properties Is, In General, NP-Hard – Unless We Measure Everything

Yan Wang[1], Oscar Galindo[2], Michael Baca[2],
Jake Lasley[2], and Vladik Kreinovich[2]
[1]School of Mechanical Engineering, Georgia Institute of Technology,
Atlanta, GA 30332-0405, USA, yan.wang@me.gatech.edu
[1]University of Texas at El Paso, El Paso, TX 79968, USA
ogalindomo@miners.utep.edu, mvbaca@miners.utep.edu
jlasley@miners.utep.edu, vladik@utep.edu

Linear elasticity: a brief reminder. A force applied to a rubber band extends it or curves it. In general, a force applied to different parts of an elastic body changes the mutual location of its points. Once we know the forces applied at different locations, we can determine the deformations – and, vice versa, we can determine the forces once we know all the deformations. In general, the dependence on forces f_i at different locations on different displacement is non-linear; however, when displacements are small, we can ignore terms quadratic or higher order in terms of ε_j and thus safely assume that the dependence of each force component f_α on all the components ε_β of displacements at difference locations is linear. Taking into account that in the absence of forces, there is no displacement, we conclude that $f_\alpha = \sum\limits_\beta K_{\alpha,\beta} \cdot \varepsilon_\beta$ for some coefficients $K_{\alpha,\beta}$. These coefficients describe the mechanical properties of the body. It is therefore desirable to experimentally determine these coefficients.

Ideal case. In the ideal case, we measure displacements ε_β and forces f_α at all possible locations. Each such measurement results in an equation which is linear in terms of the unknowns $K_{\alpha,\beta}$. Thus, after performing sufficiently many measurements, we get an easy-to-solve system of linear equations that enables us to find the values $K_{\alpha,\beta}$.

In practice, we only measure some values. In reality, we only measure displacements and forces at some locations – i.e., we know only some values ε_β. In this case, since both $K_{\alpha,\beta}$ and some values ε_β are unknown, the corresponding system of equations becomes quadratic. After sufficiently many measurements, we can still uniquely determine $K_{\alpha,\beta}$, but the reconstruction is more complex.

What we prove. How more? In this talk, we prove that the corresponding reconstruction problem is, in general, NP-hard.