How to Define "and"- and "or"-Operations for Intuitionistic and Picture Fuzzy Sets

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In the traditional fuzzy logic, we describe our degree of confidence in a statement A by a number $a \in [0,1]$, so that 0 means no confidence, 1 means full confidence, and intermediate values describe intermediate degrees of confidence.

In practice, there is often a need to estimate the degree of confidence in composite statements like $A \,\&\, B$ and $A \vee B$. There are many such statements, so it is not feasible to ask the experts about all of them. Instead, we must estimate our degree of confidence in $A \,\&\, B$ and $A \vee B$ based on our degrees of confidence a and b in the statements A and B. These estimates $f_\&(a,b)$ and $f_\vee(a,b)$ are known as "and"- and "or"-operations. The most widely used operations are $f_\&(a,b) = \min(a,b)$ and $f_\vee(a,b) = \max(a,b)$.

In the traditional fuzzy logic, we do not distinguish between the cases when we know nothing about A and when we have equally strong arguments for and against A: in both cases, we assign $a = 0.5$. To make this distinction, we can use two degrees: the degree a_+ of confidence in A and the degree of confidence a_- in $\neg A$, for which $a_+ + a_- \le 1$. In this *intuitionistic fuzzy* approach, in the first case, we have $a_+ = a_- = 0$, in the second $a_+ = a_- = 0.5$. To define "and"- and "or"-operations to intuionistic fuzzy sets, we can find $f(a,b)$ for which $a_+ + a_- \le 1$ and $b_+ + b_- \le 1$ always imply $f_\&(a_+,b_+) + f(a_-,b_-) \le 1$.

If $a_+ + a_- < 1$, this means that we do not have enough evidence for or against A. This may mean that we are still trying, or it may means that we are not interested in A at all – or it may mean that we are interested to some degree a_0, for which $a_+ + a_- + a_0 \le 1$. How do we define "and"- and "or"-operations on such *picture sets*? Again, a similar idea is to find a function $g(a,b)$ for which $a_+ + a_- + a_0 \le 1$ and $b_+ + b_- + b_0 \le 1$ always imply $f_\&(a_+,b_+) + f(a_-,b_-) + g(a_0,b_0) \le 1$.

In this talk, we use the above idea to describe possible "and"- and "or"-operations for intuitionstic and picture sets. These turn out to be the same operations that have been shown to be practically successful – but now we have a theoretical explanation for these heuristic operations.