Computing with Words – When Results Do Not Depend on the Selection of the Membership Function

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Formulation of the problem. One of the most successful ways to transform natural-language expert knowledge into computer-understandable numerical form is to use fuzzy logic. In fuzzy logic, each imprecise property like "small" is described by a *membership function* that assigns, to each possible value x, a degree $\mu(x)$ to which x is, e.g., small. The problem is that membership functions are subjective. It is therefore desirable to look for cases when the results do not depend on this subjective choice.

Continuity: known example. Intuitively, continuity means that if x' is close to x, then $y' = f(x')$ should be close to $y = f(x)$. In other words, if $x' - x$ is small, then $f(x') - f(x)$ should be small. Thus, the degree $\mu^y_{\text{small}}(f(x') - f(x))$ cannot be smaller that $\mu^x_{\text{small}}(x' - x)$. The quantities x and y may differ by scale, so $\mu^y_{\text{small}}(z) = \mu^x_{\text{small}}(K \cdot z)$, thus $\mu^x_{\text{small}}(K \cdot (f(x') - f(x)) \geq \mu^x_{\text{small}}(x' - x)$, hence $K \cdot |f(x') - f(x)| \leq |x - x'|$ and $|f(x') - f(x)| \leq K^{-1} \cdot |x' - x|$. Thus, the common sense continuity leads to the Lipschitz condition.

New examples. What if we have a relation between x and y and not a function? In this case, continuity still implies that $f(x)$ is a function.

What is the dependence of y on x and x on y are both continuous? Then, we have $|f(x') - f(x)| \leq K^{-1} \cdot |x' - x|$ and $|x' - x| \leq K \cdot |f(x') - f(x)|$, hence $|f(x') - f(x)| = K^{-1} \cdot |x' - x|$ for all x and x'. One can prove that this is only possible when $f(x)$ is linear.

What if $f(x)$ is growing? Intuitively, it means that if $x' \gg x$, then $f(x') \gg f(x)$. For any membership function for "much larger", we get $f(x') - f(x) \geq K \cdot (x' - x)$ for $x' > x$, i.e., in effect, $f'(x) \geq K$ for some $K > 0$.