Asymptotic Analysis of a Nonlinear Oscillator with Damping

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Abstract

We consider the following second order evolution equation modelling a nonlinear oscillator with damping
$u''(t) + \gamma u'(t) + Au(t) = f(t)$, (SEE)
Where A is a maximal monotone and α-inverse strongly monotone operator in a real Hilbert space H. With suitable assumptions on γ and $f(t)$ we show that the zero set of A is nonempty, if and only if (SEE) has a bounded solution and in this case we provide approximation results for elements of the zero set of A by proving weak and strong convergence theorems for solutions to (SEE) showing that the limit belongs to the zero set of A. As a discrete version of (SEE), we consider the following second order difference equation
$u_{n+1} - u_n - \alpha_n(u_n - u_{n-1}) + \lambda_n A u_{n+1} \ni f(t)$,

where A is assumed to be only maximal monotone (possibly multivalued). By using the results in [11], we prove ergodic, weak and strong convergence theorems for the sequence u_n, and show that the limit is the asymptotic center of u_n and belongs to the zero set of A. This again shows that the zero set of A is nonempty if and only if u_n is bounded. Also these results solve an open problem raised in [2], namely the study of the convergence results for the inexact inertial proximal algorithm. Our paper is motivated by the previous results in [6–11] and significantly improves upon the results of H. Attouch and P. E. Maingé [3], and F. Alvarez and H. Attouch [2].