Fourier Transform and Other Quadratic Problems under Interval Uncertainty

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Need for data processing. Computers are used to estimate the current values of physical quantities and to predict their future values – e.g., to predict tomorrow's temperature.

Need to take uncertainty into account. The inputs $x_1, \ldots, x_n$ for such data processing come from measurements (or from expert estimates). Both measurements and expert estimates are not absolutely accurate: measurement results $\widetilde{x}_i$ are, in general, somewhat different from the actual (unknown) values x_i of the corresponding quantities. Because of these differences $\widetilde{x}_i - x_i$ (called *measurement errors*), the result $\widetilde{y} = f(\widetilde{x}_1, \ldots, \widetilde{x}_n)$ of data processing is also somewhat different from the actual value of the desired quantity y — at least from the value $y = f(x_1, \ldots, x_n)$ that we would have obtained if we knew the exact values x_i of the inputs.

Need for interval uncertainty. In many practical situations, the only information that we have about measurement uncertainty is the upper bound Δ_i on the absolute value of each measurement error. In such situations, if the measurement result is $\widetilde{x}_i$, then all we know about the actual value x_i of the corresponding quantity is that this value is in the interval $[\widetilde{x}_i - \Delta_i, \widetilde{x}_i + \Delta_i]$.

Under such interval uncertainty, it is desirable to know the range of possible value of y.

Interval uncertainty: what is known. In general, computing such a range is NP-hard already for quadratic functions $f(x_1, \ldots, x_n)$. Recently, a feasible algorithm was proposed for a practically important quadratic problem — of estimating the absolute value (modulus) of Fourier coefficients.

What we do. In this talk, we show that this feasible algorithm can be extended to a reasonable general class of quadratic problems.