Decision Making Under Uncertainty: Case When We Only Know an Upper Bound or a Lower Bound

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Formulation of the problem. In investment, when a person knows the exact monetary consequence of each action, he/she naturally selects an action with the largest possible gain. In practice, we usually know the consequences only with some uncertainty. For example, instead of the exact gain value, the whole set S of different possible gain values are consistent with our knowledge. How should we then make a decision? What is the equivalent price $v(S)$ that we are willing to pay to participate in the corresponding action?

For example, we may know the lower bound a and the upper bound on the gain. In this case, the set S is the interval $[a, b]$. Alternatively, we may only know the lower bound, in which case $S = [a, \infty)$ or only the upper bound, in which case $S = (-\infty, b]$.

How this problem is solved if we know both bounds. If we are willing to pay $v(S)$ for the set S, then for set S *and* a fixed amount c, we are willing to pay $v(S) + c$. In this joint offer, the set of possible outcomes is $S + c \stackrel{\text{def}}{=} \{s + c : s \in S\}$. Thus, we should have $v(S + c) = v(S) + c$; this is called *shift-invariance*. Another idea is that the transformation $S \mapsto v(S)$ should not depend on the choice of the monetary unit: if we select pesos instead of dollars, we should get the same equivalent value. In precise terms, this means $v(\lambda \cdot S) = \lambda \cdot v(s)$, where $\lambda \cdot S \stackrel{\text{def}}{=} \{\lambda \cdot s : s \in S\}$; this property is known as *scale-invariance*. The third idea is that participation in two independence actions, with sets S_1 and S_2, is equivalent to participation in a single action with the result $S_1 + S_2 = \{s_1 + s_2 : s_1 \in S_1 \,\&\, s_2 \in S_2\}$. These are two ways of representing the same situation, so we should have $v(S_1 + S_2) = v(S_1) + v(S_2)$; this property is known as *additivity*. For interval uncertainty, additivity implies Hurwicz formula $v([a, b]) = \alpha \cdot b + (1 - \alpha) \cdot a$ for some $\alpha \in [0, 1]$. The same formula emerges if we assume shift- and scale-invariance.

Case of infinite intervals: new results. It turns out that additivity implies $v([a, \infty)) = k \cdot a$ for some $k \geq 1$. A similar formula emerges if we assume scale-invariance. If we assume shift-invariance, then we get $v([a, \infty)) = a + a_0$ for some $a_0 \geq 0$. Similar results emerge for $S = (-\infty, b]$.