Interregular Polygons and the Discrete Dynamics of Restricted Polygon Stacking

Adan J. Mireles

Department of Metallurgical and Materials Engineering

The University of Texas at El Paso, El Paso, TX

Abstract

We introduce and provide characterization of a new class of convex polygons termed *interregular* polygons. Playing the role of geometric intermediates, the latter can be interpreted as regular polygons with non-integer number of sides. We distinguish between two kinds of interregular polygons: β_1- and β_2-interregular polygons. Being *cyclic* or *tangential*, these polygons have different number of congruent interior and central angles, and, therefore, give rise to different formulas for calculating their area and perimeter. We prove a duality result by showing that each interregular polygon of one kind has a "dual" polygon which belongs to the opposite kind. Additionally, we discuss the connection of interregular polygons with L. Schläfli's regular star polygons. We also study a set of sequences which involve the vertical stacking of infinitely many regular or interregular polygons under a restricting curve, i.e. a polygonal tower with limited height. Such curve is represented as a function $R(x)$ which is even, and both $R'(x)$ and $R''(x)$ are negative in the interval $0 < x < a$, where a is a root of $R(x)$. A guided construction and software implementation of the sequences is provided, which rely on unique recursive functions and make use of techniques from Cartesian geometry. An example of a polygonal tower is analyzed— the restricting curve is taken as the top half of a circle of radius 1 and the stacked polygon as a square. For this instance, it is shown that the area of the fitted polygons converges to 0, and that their side lengths quadratically converge to 0 (their negative logarithmic value doubles with each iteration). Finally, a recursive function is provided to approximate the side length of the n-th stacked square under the curve.

Keywords: polygon characterization; interregular polygons; non-integer sides; star polygons; polygon stacking; polygon sequences.