Hunting Habits of Predatory Birds: Explanation of an Empirical Formula

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Formulation of the Problem. Predatory birds are an important part of an ecosystem. Like all predators, they help maintain the healthy balance in nature. This balance is very delicate, unintended human interference can disrupt it. To avoid such disruption, it is important to study the hunting behavior of predatory birds. This behavior is cyclic. Most predatory birds like owls spend some time waiting for the prey, and then either attack or jump to a new location. For the same bird, waiting time w changes randomly from one cycle to another. Researchers recently found how the probability $f(t) \stackrel{\text{def}}{=} \text{Prob}(w \geq t)$ that the waiting time is $\geq t$ depends on t: $f(t) \approx A \cdot t^{-a}$. How can we explain this empirical observation?

We Need a Family of Functions. Some birds tend to wait longer, some tend to wait less. So, we cannot have a single formula that would cover all the birds of the same species. We need a family of functions $f(t)$. The simplest family if when we fix some function $F(t)$ and consider all possible functions of the type $C \cdot F(t)$. What family should we choose?

Invariance. The numerical value of waiting time depends on the selection of the measuring unit. If we replace the original measuring unit with the one which is λ times smaller, all numerical values multiply by λ: $t \mapsto \lambda \cdot t$. It looks like there is no preferable measuring unit. So, it makes sense to assume that the family $\{C \cdot F(t)\}_C$ should remain the same if we change the measuring unit. This implies, in particular, that for every $\lambda > 0$, the function $F(\lambda \cdot t)$ should belong to the same family. Thus, for every $\lambda > 0$, there exists a constant C depending on λ for which $F(\lambda \cdot t) = C(\lambda) \cdot F(t)$.

It is known that every measurable solution to the functional equation $F(\lambda \cdot t) = C(\lambda) \cdot F(t)$ has the form $r(t) = A \cdot t^a$. This is exactly the empirical probability distribution – it is only one which does not depend on the selection of the measuring unit for time.

How to Prove the Result about the Functional Equation. This result is easy to prove when the function $F(t)$ is differentiable. Suppose that $F(\lambda \cdot t) = C(\lambda) \cdot F(t)$. If we differentiate both sides with respect to λ, we get $t \cdot F'(\lambda \cdot t) = C'(\lambda) \cdot F(t)$. In particular, for $\lambda = 1$, we get $t \cdot F'(t) = a \cdot F(t)$, where $a \stackrel{\text{def}}{=} C'(1)$, so $t \cdot \dfrac{dF}{dt} = a \cdot F$. We can separate the variables if we multiply both sides by $\dfrac{dt}{t \cdot F}$, then we get $\dfrac{dF}{F} = a \cdot \dfrac{dt}{t}$. Integrating both sides of this equality, we get $\ln(F) = a \cdot \ln(t) + C$. By applying $\exp(x)$ to both sides, we get $F(t) = \exp(a \cdot \ln(t) + C) = A \cdot t^a$, where $A \stackrel{\text{def}}{=} e^C$.