Need for Optimal Distributed Measurement of Cumulative Quantities Explains the Ubiquity of Absolute and Relative Error Components

Hector Reyes, Jeffrey Escamilla, Ethan D. Kish,
Aaron D. Brown, and Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso
El Paso, Texas 79968, USA
hareyes2@miners.utep.edu, jescamilla2@miners.utep.edu,
edkish@miners.utep.edu, adbrown3@miners.utep.edu,
vladik@utep.edu

Need for distributed measurements. In many practical situations, we are interested in estimating the value x of a cumulative quantity; e.g, we want to estimate the amount of oil in a given area. Measuring instruments usually measure quantities in a given location. Thus, they measure local values $x_1, \ldots, x_n$ that together form the desired value $x = x_1 + \ldots + x_n$. So, a natural way to produce an estimate $\widetilde{x}$ for x is to place measuring instruments at several locations, to measure the values x_i in these locations, and to add up the results $\widetilde{x}_1 + \ldots + \widetilde{x}_n$ of these measurement: $\widetilde{x} = \widetilde{x}_1 + \ldots + \widetilde{x}_n$.

Need for optimal planning. Usually, we want to reach a certain estimation accuracy. To achieve this accuracy, we need to plan how accurate the deployed measurement instruments should be. Use of accurate measuring instruments is often very expensive, while budgets are usually limited. It is therefore desirable to come up with the deployment plan that would achieve the desired overall accuracy within the minimal cost. This implies, in particular, that the resulting estimate should not be more accurate than needed. Indeed, this would mean that we could use less accurate (and thus, cheaper) measuring instruments.

In this talk, we provide a condition under which such optimal planning is possible, and the corresponding optimal planning algorithm. The resulting condition explains why usually, measuring instruments are characterized by their absolute and relative accuracy.

Our analysis. We want to measure x with some accuracy $\delta(x)$ by using measuring instruments with accuracy $\Delta(x)$. The approximation error $\Delta x \stackrel{\text{def}}{=} \widetilde{x} - x$ is equal to sum $\Delta x = \Delta x_1 + \ldots + \Delta x_n$ of measurement errors $\Delta x_i \stackrel{\text{def}}{=} \widetilde{x}_i - x_i$. In many practical situations, we only know the upper bound $\Delta(x_i)$ on each measurement error In this case, the approximation error is bounded by the sum of the bounds, so we must have $\delta(x) = \Delta(x_1) + \ldots + \Delta(x_n)$ once $x = x_1 + \ldots + x_n$.

Differentiating both sides of the equation $\delta'(x_1 + x_2 + \ldots + x_n) = \Delta'(x_1)$. The right-hand side does not depend on x_2, so the left-hand side should not depend on x_2 either. Thus, $\delta'(x) = \text{const}$, and therefore, $\delta(x)$ is a linear function. For $x_i = x/n$, we have $\delta(n \cdot x_1) = n \cdot \Delta(x_1)$, so we conclude that $\Delta(x)$ is also a linear function: $\Delta(x) = a + b \cdot x$ for some a and b.

This is in line with the usual description of uncertainty, where a is called an absolute error component (e.g., $\pm 0.1V$), and $b \cdot x$ is called a relative error component (e.g., $\pm 1\%$). Thus, we have a theoretical explanation for this usual description.