What If an Interval Is Given by an Expert Who Is Not Sure?

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Formulation of the problems. In many real-life situations, we ask experts to estimate the probability p of a certain event – e.g., of an earthquake of certain strength. Expert estimates are, of course, approximate, experts understand this, so they can usually provide not only the numerical estimate $\widetilde{p}$, but also some information about the accuracy of their estimate. This information often comes as a bound Δ on the estimation error $\Delta p \stackrel{\text{def}}{=} \widetilde{p} - p$. For example, an expert may say that the probability is $80\% \pm 10\%$. In this case, what the expert provides is an *interval* of possible probabilities: in the above example, the interval $[70, 90]$, and in general, the interval $[\underline{p}, \overline{p}] = [\widetilde{p} - \Delta, \widetilde{p} + \Delta]$.

Here are the two related problems:

1. Sometimes, in addition to the interval, the expert also provides a degree of confidence c in his/her estimate – e.g., 80%. How can we combine this information with the interval?

2. And what if the expert cannot provide the degree of confidence? How can we estimate this degree?

Possible solution to the first problem. With confidence c, the expert believes that the actual probability is in the interval $[\underline{p}, \overline{p}]$. This means that with the remaining probability $1 - c$, the expert does not have any information, i.e., the probability can be anywhere in $[0, 1]$. The overall expected probability is thus $c \cdot p_1 + (1 - c) \cdot p_2$, where $p_1 \in [\underline{p}, \overline{p}]$ and $p_2 \in [0, 1]$. According to interval computations (see, e.g., [1, 2, 3, 4]), the set of all such values is $c \cdot [\underline{p}, \overline{p}] + (1 - c) \cdot [0, 1] = [c \cdot \underline{p}, 1 - c \cdot (1 - \overline{p})]$.

Possible solution to the second problem. Suppose that we have an even more reliable (but more difficult-to-consult) expert whose opinion we treat as accurate. On several examples $k = 1, \ldots, n$, we ask the opinions of both experts, and get intervals $[\underline{p}_k, \overline{p}_k]$ from the expert whom we are evaluating and $[\underline{r}_k, \overline{r}_k]$ from the more reliable expert. We then expect that $c \cdot \underline{p}_k \approx \underline{r}_k$ and $1 - c \cdot (1 - \overline{p}_k) \approx \overline{r}_k$. Thus, we can find c by applying, e.g., the Least Squares approach to this system of approximate equalities, i.e., by finding c for which

$$\sum_{k=1}^{n} \left[(c \cdot \underline{p}_k - \underline{r}_k)^2 + (1 - c \cdot (1 - \overline{p}_k) - \overline{r}_k)^2 \right] \to \min_c .$$

References

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