Title

Unconditional energy stable IEQ-FEMs for the Cahn-Hilliard-Navier-Stokes equations

Authors

Yaoyao Chen[†], Dongqian Li[‡], Yin Yang[‡], Peimeng Yin[§]

[†] School of Mathematics and Statistics, Anhui Normal University, Wuhu, Anhui 241000, PR China, [‡] School of Mathematics and Computational Science, Xiangtan University, Xiangtan, Hunan 411105, PR China, [‡] School of Mathematics and Computational Science, Xiangtan University, Xiangtan, Hunan 411105, PR China, [§] Department of Mathematical Sciences, The University of Texas at El Paso, El Paso, TX 79968, USA

Abstract

We propose several unconditionally energy stable invariant energy quadratization (IEQ) finite element methods (FEMs) to solve the Cahn-Hilliard-Navier-Stokes (CHNS) equations. The time discretization of these IEQ-FEMs is based on the first- and second-order backward differentiation methods. The intermediate function introduced by the IEQ approach is positioned in different function spaces: the continuous function space, and a combination of the continuous function and finite element spaces. These methods offer distinct advantages. Consequently, we propose a new hybrid IEQ-FEM that combines the strengths of both schemes, offering computational efficiency and unconditional energy stability in the finite element space. We provide rigorous proofs of mass conservation and energy dissipation for the proposed IEQ-FEMs. Several numerical experiments are presented to validate the accuracy, efficiency, and solution properties of the proposed IEQ-FEMs.