Dual-pairing summation-by-parts framework for nonlinear conservation laws

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Abstract

Robust and stable high order numerical methods for solving partial differential equations are attractive because they are efficient on modern and next generation hardware architectures. However, the design of provably stable numerical methods for nonlinear hyperbolic conservation laws pose a significant challenge, as initial attempts often result in crashes due to compounding numerical errors or the presence of undesirable numerical oscillations which can pollute numerical simulations everywhere. Desirable high order accurate methods for nonlinear PDEs must be robust (provably stable) and preserve several important invariants present in the system.

We present the dual-pairing (DP) and upwind summation-by-parts (SBP) finite difference (FD) framework for accurate and robust numerical approximations of nonlinear conservation laws. The DP SBP FD operators are a dual-pair of backward and forward FD stencils which together preserve the SBP property. In addition, the DP SBP FD operators are designed to be upwind, that is they come with some built-in dissipation everywhere, as opposed to discontinuous Galerkin methods which can only induce dissipation through numerical fluxes acting at element interfaces. We combine the DP SBP operators together with skew-symmetric and upwind flux splitting of nonlinear hyperbolic conservation laws. Our semi-discrete approximation is conservative and provably entropy-stable for arbitrary nonlinear hyperbolic conservation laws. We give specific examples using the in-viscid Burger's equation, nonlinear shallow water equations and compressible Euler equations of gas dynamics. Numerical experiments are presented to verify accuracy and demonstrate the robustness of our numerical framework.

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