Visualization and Reliable Computation of Roots in Complex Functions

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Complex numbers, written as $a + ib$, where a is the real part and b is the imaginary part, were created to represent the square root of negative numbers. They are essential for solving problems in fields like Electrical Engineering, Number Theory, Control Theory, Quantum Mechanics, Electromagnetism, and Computer Science. Finding the roots of complex functions in these fields can be challenging due to the multiplicity of the roots, complex plane behavior, analytic properties, and interactions with other functions.

To make this process easier, we have developed a way to visualize these functions on the complex plane using colors. The colors represent the phase or angle of the complex output, and where these colors merge, we find the roots or poles. We differentiate between roots and poles by their movement: When the colors merge and move counterclockwise, it indicates a root. When the colors merge and move clockwise, it indicates a pole. This is because the pole's behavior is represented $e^{-\theta}$, which causes the phase to move in the opposite direction compared to a root. Once we identify the root boundaries, we use an interval solver to accurately find the roots.

Discovering the locations of the roots of a complex function is crucial as it provides insight into the behavior, structure, and solutions of real-world and mathematical problems. With this knowledge, we can tackle bigger challenges, such as the Riemann Hypothesis, a major unsolved problem in mathematics.

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