Fair Economic Division: How to Modify Shapley Value to Take Into Account that Different People Have Different Productivity

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Shapley value: a brief reminder. Let us assume that a group $N \stackrel{\text{def}}{=} \{1, \ldots, n\}$ of n people jointly gets some benefit $v(N)$. What is the fair way to distribute this benefit between the participants, i.e., to assign values $x_1, \ldots, x_n$ whose sum is $v(N)$? To make this distribution, it is important to know what is the contribution of each participant. This can be described by providing, for each subset $S \subset N$, a value $v(S)$ that people from S would have gained if they acted on their own, without help of others. So, we get a function $S \mapsto v(S)$ that characterizes the situation. A Nobelist Lloyd Shapley found out that under reasonable conditions, there is only one way to assign the distribution $x = (x_1, \ldots, x_n)$ to each such function $v(S)$.

Shapley's first condition is symmetry: if two participants i and j contribute equally, i.e., if the values $v(S)$ do not change when we swap i and j, then these participants should get equal amounts: $x_i = x_j$. Shapley's second condition is that if a person i is not contributing, i.e., if $v(S \cup \{i\}) = v(S)$ for all S, then we should have $x_i = 0$. Shapley's third condition is additivity: if we have two situations $u(S)$ and $v(S)$, then we can either consider them separately or view them as a single situation with gain $w(S) = u(S) + v(S)$. The outcome should not depend on how we view this, so we should have $x_i(w) = x_i(u) + x_i(v)$.

Comment. In this talk, we will only deal with economic applications of Shapley value. It should be mentioned that Shapley value is now also actively used in machine learning, to find the importance x_i of each of n features based on the effectiveness $v(S)$ of solving the problem when we only use features from the set S.

Why go beyond Shapley value. Symmetry makes perfect sense if all participants are equally productive. In reality, people have different productivity: e.g., some programmers are several times more productive than others. If we naively apply Shapley value to compute each person's bonus, more productive participants will get the exact same amount as less productive ones, which is not fair. It is therefore desirable to take productivity w_i of each participant into account. In other words, we need to determine x_i based on both $v(S)$ and the values w_i.

What we propose. If i's productivity is twice larger than j's, this means that the company can replace i with two less productive workers and get the same result. After this replacement, all participants have the same productivity, so to this replaced situation, we can apply symmetry and get Shapley value – and then assign to i the sum of bonuses that Shapley value recommends for his/her two replacements.

It turns out that after properly formalizing this idea, we have a unique mapping $x_i(v, w)$. When all the weights are equal, then, as expected, this formula becomes the usual Shapley value.