

From Nonlinearity to Linearity: Solving Riccati Equations

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Abstract

This research investigates the transformation of Riccati equations into second-order linear differential equations. By introducing a shifted variable, we reformulate nonlinear Riccati equations to allow derivation of corresponding second-order linear ODEs. Several propositions establish conditions under which these transformations hold and enable analytical solutions using integrating factors. We explore the structure of adjoint equations and assess the self-adjointness of Riccati and transformed systems using variational derivatives. Examples demonstrate the effectiveness of this method in solving complex ODEs with variable coefficients. Additionally, the study reveals a systematic approach for constructing solutions to Riccati equations from the solutions of their linear counterparts. This approach links nonlinear and linear systems, highlighting the utility of classical techniques in modern differential equation analysis and offering new insight into solvability conditions and solution methods.

Keywords: Differential Equations, Riccati Equations, Integrating Factor, Adjoint, Self-adjoint

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