Basic Graph Theory and Quantum Analogs

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Graph theory is an area of discrete mathematics having diverse applications ranging from computer science to physics to linguistics. Its origins lie in the famous Seven Bridges of Königsberg problem of the 18th century and the subject continues to be an active area of research to this day. A graph is a pair (V, E) where V is a set whose elements are called vertices and E is a set of unordered pairs of vertices which are called edges. The edges of the graph define a symmetric relation $\sim$ on the set of vertices defined by $a \sim b \iff \{a, b\} \in E$. If $a \sim b$ we say that a is *adjacent* to b. Because the edge set consists of unordered pairs, we are only considering *undirected* graphs. Two basic properties of graphs are those of regularity and connectedness. A graph is *d-regular* if every vertex is adjacent to d vertices for some non-negative integer d. A graph is connected if there is a path from any vertex to another. Put more formally, a graph is connected if, for any two distinct vertices v and u, there exists a sequence of vertices beginning at v and ending at u such that any two consecutive elements of the sequence form an edge. These definitions serve as a guide for defining quantum graphs and quantum versions of regularity and connectedness.